A METHOD OF DETERMINING THE CONSTANTS IN THE BIMODAL FOURTH DEGREE EXPONENTIAL FUNCTION

By

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In a paper in this Journal[1] the present writer has discussed some of the mathematical properties of a class of definite integrals which arise in the study of the frequency function

$$(1) \qquad y = e^{-a^2(x^4+p_1x^3+p_2x^2+p_3x+p_4)}, \quad a \neq 0.$$

This function defines the system of frequency curves for which the method of moments is the best method of fitting[2]—i.e. best in the sense of maximum likelihood—and this fact gives importance to its study. The curves are typically bimodal, the nature and location of the modes being given by the roots of the equation

$$(2) \qquad 4x^3+3p_1x^2+2p_2x+p_3 = 0.$$

The first problem which arose was that of finding an expression for the value of the definite integral

$$(3) \qquad I_o = \int_{-\infty}^{\infty} e^{-a^2(x^4+p_1x^3+p_2x^2+p_3x+p_4)}\,dx.$$

If x is replaced by $x - \frac{p_1}{4}$ this integral becomes

$$(4) \qquad I_o = \int_{-\infty}^{\infty} e^{-a^2(x^4+px^2+qx+r)}\,dx,$$

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[1] On the system of curves for which the method of moments is the best method of fitting. Vol. IV, No. 1, Feb. 1933, p. 1.

[2] R. A. Fisher, On the mathematical foundations of theoretical statistics, Philosophical Transactions of the Royal Society of London, vol. 222, series A (1921), p. 355.