

ON SAMPLING FROM COMPOUND POPULATIONS*

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Introduction.

The decided asymmetry or the multimodality of certain frequency distributions may have prompted the idea of the possibility of the existence of frequency curves, apparently single in character, but which, on further investigation, might be shown to be actually composite. In other words, apparently homogeneous material may prove to be heterogeneous, or divisible into two or more distinct homogeneous groups.

The above ideas lead naturally to the problem of dissecting a compound frequency function into its various components. Karl Pearson¹ successfully solved such a problem, using the method of moments, on the assumption that the compound parent population was composed of two normal components. Each component curve has three parameters, the mean (or position of axis), the standard deviation, and the area (or total frequency). One requires therefore, six relations between the parameters of the given compound frequency curve, and those of its two components, in order to determine six unknowns. The ultimate solution of the problem turns on the determination of the zeros of a nonic equation, the location of whose real roots is obtained, to successive approximations, by means of the so-called Sturm's functions.

The dissection problem was taken up later, first in a paper by Charlier,² then in a joint paper by Charlier and Wicksell³ who

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¹ On the dissection of frequency curves into normal curves. Karl Pearson. *Phil. Trans. Roy. Soc. Lond.* Vol. 185, Pt. 1, pp. 71-110. 1894A.

² Researches into the theory of probability. C. V. L. Charlier. *Meddelanden frau Lunds Astron. Observ.* Sec. 2. Bd 1. 1906.

³ On the dissection of frequency functions. C. V. L. Charlier, and S. D. Wicksell. *Arkiv. fur Matematik. Astron. och Fysik.* (Meddelande) Band 18. No. 6. 1923.