

ON THE ELIMINATION OF SYSTEMATIC ERRORS DUE TO GROUPING

By
JOHN R. ABERNETHY

In the calculation of the moments of a frequency distribution it is often desirable, or even necessary, to consider not the distribution itself but another derived from it by certain groupings. As a first approximation to the moments of the original distribution we take the corresponding moment of the grouped distribution. But this first approximation is not satisfactory, and it is necessary to obtain some method for the elimination of part of the error committed in replacing the moments of the original distribution by the corresponding moments of the grouped distribution.

This problem was first discussed by W. F. Sheppard in a paper : *On the Calculation of the most Probable Value of Frequency-Constants, for Data arranged according to Equidistant Divisions of a Scale.*¹ If we denote the n -th moment of the original distribution by μ_n and the n -th moment of the grouped distribution by ν_n , we will have Sheppard's corrections in the form :

$$\begin{aligned}\mu_1 &= \nu_1 = 0, \\ \mu_2 &= \nu_2 - \frac{1}{12}, \\ \mu_3 &= \nu_3, \\ \mu_4 &= \nu_4 - \frac{1}{2} \nu_2 + \frac{7}{240}, \\ \mu_5 &= \nu_5 - \frac{5}{6} \nu_3, \qquad \text{etc.}\end{aligned}$$

As pointed out by Karl Pearson² the hypotheses under which these formulae have been obtained are: (a) that Taylor's theorem

¹ Proceedings London Mathematical Society, Vol. 29, p. 353-380.

² *On an elementary proof of Sheppard's formulae for correcting raw moments and other allied points*, editorial in *Biometrika*, Vol. 3, p. 308-312.