QUADRATURE OF THE NORMAL CURVE

By

E. R. Enlow

There are three formulas for the calculation of areas under the normal probability curve, only two of which seem to be generally recognized in American statistical circles. Herewith is presented an outline of the mathematical development of these three formulas and a determination of the bounds of practical utility of each.

The well-known equation for the normal curve,

$$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

may be expanded into the series

$$y = \frac{N}{\sigma\sqrt{2\pi}}\left[1 - \left(\frac{x}{\sigma\sqrt{2}}\right)^2 + \frac{1}{\underline{|2}}\left(\frac{x}{\sigma\sqrt{2\pi}}\right)^4 - \frac{1}{\underline{|3}}\left(\frac{x}{\sigma\sqrt{2\pi}}\right)^6 + \cdots\right]$$

(Ref. 3)

by means of Maclaurin's Theorem. (See any good calculus text-book.) (7) This expansion is readily accomplished by making the substitution

$$t = \frac{x}{\sigma\sqrt{2}}$$

so that

$$e^{-\frac{x^2}{2\sigma^2}} = e^{-t^2}$$

and

$$f(t) = e^{-t^2}$$

The process of successive differentiation is quite lengthy, since every other term differentiated becomes zero and therefore 2n terms in the Maclaurin series are required to produce n terms in the new series. After the expansion has been carried to five or six terms, a regular law of formation becomes evident from inspection of the new series

$$e^{-t^2} = 1 - t^2 + \frac{1}{\underline{|2}} t^4 - \frac{1}{\underline{|3}} t^6 + \frac{1}{\underline{|4}} t^8 - \cdots$$