THE STANDARD ERROR OF ANY ANALYTIC FUNCTION OF A SET OF PARAMETERS EVALUATED BY THE METHOD OF LEAST SQUARES

By

WALTER A. HENDRICKS,
Bureau of Animal Industry,
U. S. Department of Agriculture, Washington, D. C.

After fitting a curve to a set of data by the method of least squares, it is occasionally necessary to use the resulting values of some or all of the parameters of the curve in further calculations. Since the estimates of the values of the parameters obtained from a particular set of data are subject to errors of sampling, it follows that the result of any calculation involving those values of the parameters will have a certain standard error. Since the estimated values of the parameters are not independent of each other, the familiar formulas based on the assumption of independence should not be used for the purpose of calculating this standard error from the standard errors of the parameters themselves. The correct approach to the problem involves little more than an application of the methods presented by Schultz (1930) in his excellent paper describing the method of calculating the standard error of a particular function of the parameters, viz., the same function which was used in evaluating the parameters.

Let $y = \varphi(\lambda_1, \lambda_2, \cdots, \lambda_k)$ be an analytic function involving the k parameters, λ_i. This function may not be linear with respect to the parameters, so that if the parameters are to be evaluated by the method of least squares, we have in the general case a function of the form:

$$(1) \qquad y = \varphi(\lambda_{1_0}, \lambda_{2_0}, \cdots, \lambda_{k_0}) + \frac{\partial \varphi}{\partial \lambda_{1_0}} \Delta\lambda_1 + \cdots + \frac{\partial \varphi}{\partial \lambda_{k_0}} \Delta\lambda_k + \cdots$$