THE POINT BINOMIAL AND PROBABILITY PAPER

By Frank H. Byron[1]

1. An approximation to the sum of a number of consecutive terms of the point binomial may be found graphically and quite expeditiously by means of so-called "probability paper." This paper is ruled so that the (x, y) graph of the equation of the integral of the normal curve

$$y = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x}{2}} dx \tag{1}$$

is a straight line. Let the successive terms of the point binomial be represented as follows:

$$(p + q)^n = u_0 + u_1 + \cdots + u_t + \cdots + u_n, \tag{2}$$

where $u_t = {}_nC_t p^{n-t} q^t$ and $p \geqq q$. Then the (x, y) graph of the equation,

$$y = \sum_{i=0}^{t} u_i, \qquad t + \tfrac{1}{2} = x, \tag{3}$$

i.e., of the sum of first $(t + 1)$ terms of this point binomial, is, in all but extreme cases, a set of points lying on a gently turning curve, so gently that its form may be represented closely by two straight lines, each passing through the median point as will be explained in the next section. As paper of this sort is readily obtainable, and as this method yields as great accuracy as is really useful in many problems, it is suggested that its use ought to be quite general.

2. **Sheppard's Corrections.** The formulae for the moments of the point binomial, mean $= qn$, $\sigma^2 = pqn$, are exact without any corrections such as are used for grouped material. This fact has led us all (apparently) to assume that in fitting the curve to the point binomial one would get a better fit by equating the moments of the curve to the uncorrected moments of the point binomial rather than to the corrected moments. The studies made in connection with the preparation of this paper show that when the purpose is to equate areas to sums of terms the corrected moments should be used. The theoretical basis for this conclusion is as follows:

To simplify the argument let us suppose that one were seeking that curve of Charlier type,

$$F(x) = c_0\phi_0(x) + c_1\phi_1(x) + \cdots c_4\phi_4(x), \tag{4}$$

[1] With the assistance of Burton H. Camp.