SOME INTERESTING FEATURES OF FREQUENCY CURVES

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Introduction

It is well known that in the normal error curve the points of inflection are equidistant from the mode. However it has never been pointed out that this is also a characteristic of all of the bell-shaped Pearson Frequency Curves. This fact can be most easily shown by placing the mode at the abscissa $x = 0$.

Many rough checks have been developed for use in applying the Theory of Least Squares. The second part of this paper develops a rough check on the computation for use when fitting a Pearson Frequency Curve to a set of observations. No rough checks on computation are given in textbooks on Pearson's Frequency Curves.

At present it is customary to follow a separate procedure for each Type of curve when computing the constants of a Pearson Frequency Curve. The third part of this paper shows how a single system may be followed for all Types. A single procedure is very desirable in order that the rough check of Part 2 may be quickly applied.

Part 1. Points of Inflection

Perhaps nothing brings out the limitations of the bell-shaped Pearson Curves in a more striking manner than a discussion of their points of inflection. In dealing with frequency curves it is well known that any curve can be fitted to a given distribution and that the real problem in curve fitting is the selection of a curve. Figures 1, 2, and 3 illustrate three hypothetical histograms. All three of these histograms are bell-shaped yet none of them will be closely fitted by any of the Pearson Curves. The reasons will be pointed out presently.

The differential equation from which Pearson derived his system of frequency curves is

$$\frac{dy}{dx} = \frac{y(x - P)}{b_2x^2 + b_1x + b_0}.$$

By putting $x - P = X$, i.e. by placing the mode at the abscissa $X = 0$, this differential equation may be written:

$$\frac{dy}{dX} = \frac{yX}{\pm B_2X \pm B_1X + B_0}$$

where the $+$ or $-$ sign is taken according to the type of the curve. (It will be shown later that the constant term of the denominator must be less than zero.)