A METHOD FOR DETERMINING THE COEFFICIENTS OF A CHARACTERISTIC EQUATION

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For the characteristic equation

$$\begin{vmatrix} a_{11} - x & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} - x \end{vmatrix} \equiv (-1)^n(x^n - c_1x^{n-1} + c_2x^{n-2} - \cdots + c_n) \tag{1}$$

$$\equiv (x - \alpha_1)(x - \alpha_2) \cdots (x - \alpha_n)$$

it is well known that

$$c_i = A_i$$

where A_i is the sum of all i^{th} order co-axial minors of the determinant

$$A = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}. \tag{2}$$

If n exceeds 3 or 4, the process of calculating all possible principal minors is very cumbersome.

But another more systematic method of calculating the c's may be adopted. Suppose we define

$$A^p = \begin{vmatrix} a_{11}^{(p)} & \cdots & a_{1n}^{(p)} \\ \cdots & \cdots & \cdots \\ a_{n1}^{(p)} & \cdots & a_{nn}^{(p)} \end{vmatrix} \tag{3}$$

and

$$\sum_1^n \alpha_i^{(p)} = S_p. \tag{4}$$

It may be proved[1] that

$$S_p = \sum_1^n a_{ii}^{(p)}. \tag{5}$$

But from Newton's identities[2] we have

$$S_p + c_1S_{p-1} + c_2S_{p-2} + \cdots + c_{p-1}S_1 + pc_p = 0. \tag{6}$$

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[1] Muir, L. & Metzler, W. H., "A Treatise on the Theory of Determinants," p. 606, ¶ 650 and 651.

[2] Dickson, L. E., "First Course in the Theory of Equations," p. 134, ¶ 106.