A NOTE ON SHEPPARD’S CORRECTIONS

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In this note we shall derive a simple relation between the characteristic function of the grouped distribution and the characteristic function of the original continuous distribution, assuming that the frequency curve has high contact with the x-axis at both ends.

If we set \( p_s = \int_{x_s - \frac{w}{2}}^{x_s + \frac{w}{2}} f(x) \, dx \), then the characteristic function of the grouped distribution is given by

\[ \psi(t) = \sum e^{itz} p_s, \]

where \( i = \sqrt{-1} \). Replacing \( p_s \) by its value as given above, we have

\[ \psi(t) = \sum e^{itz} \int_{x_s - \frac{w}{2}}^{x_s + \frac{w}{2}} f(x) \, dx \]

\[ = \sum e^{itz} \int_{-\frac{w}{2}}^{\frac{w}{2}} f(x + x_s) \, dx \]

\[ = \int_{-\frac{w}{2}}^{\frac{w}{2}} dx \sum e^{itz} f(x + x_s) \]

\[ = \sum e^{itz} f(x_s) \int_{-\frac{w}{2}}^{\frac{w}{2}} e^{-itz} \, dx. \]

There is no difficulty about justifying the inversion of the order of integration and summation.

Because of the assumption of high-contact with the axis of \( x \) at both ends of the frequency curve, we have

\[ \varphi(t) = \int e^{itz} f(x) \, dx = w \sum e^{itz} \varphi(x_s) \]

so that

\[ \psi(t) = \frac{2}{wt} \sin \frac{tw}{2} \varphi(t). \]