ON THE POSTULATE OF THE ARITHMETIC MEAN

By Richmond T. Zoch

Introduction

Suppose n observations have been made of an unknown quantity. It is desired to know the most probable value of the unknown. When Gauss gave his development of the so-called *Normal Law of Error*, he assumed that *the Arithmetic Mean of the n observations is the most probable value.* The question arises: Can this postulate be justified?

In the excellent book, entitled "Calculus of Observations," by Whittaker and Robinson[1] there is given a proof which purports to deduce the postulate of the Arithmetic Mean from assumptions of a more elementary nature. This proof is not correct.

Since this book has had wide circulation, it is believed that the errors in this proof should be called to the attention of the users of the book. The present paper has been prepared for this purpose. The first part of this paper points out the questionable features of the proof given in Whittaker and Robinson's book. The second part gives some critical comments on the original sources from which Whittaker and Robinson obtained their proof.

Part 1

The assumptions on which Whittaker and Robinson based their proof of the postulate of the Arithmetic Mean are:

Axiom I. The differences between the most probable value and the individual measures do not depend on the position of the null-point from which they are reckoned.

Axiom II. The ratio of the most probable value to any individual measure does not depend on the unit in terms of which the measures are reckoned.

Axiom III. The most probable value is independent of the order in which the measurements are made, and so is a symmetric function of the measures.

Axiom IV. The most probable value, regarded as a function of the individual measures, has one-valued and continuous first derivatives with respect to them.

It is fairly easy to show that if the Arithmetic Mean is the most probable value, then the above four axioms follow as conclusions. The converse, viz. if the above four axioms be assumed then the Arithmetic Mean is the most probable value, however, is not true. That is to say the above assumptions are

[1] The Calculus of Observations by E. T. Whittaker and G. Robinson, Blackie & Son, Ltd., London (1929), pp. 215–217.