FUNDAMENTALS OF THE THEORY OF INVERSE SAMPLING[1]

By Ching-Lai Shen

Part I. Introduction[2]

Section I. Statistical Concepts of the Theory of Sampling

One of the chief objects in statistics is to form a judgment of a very large statistical universe, known as a parent population, by means of a study of a part or sample thereof, which is drawn at random. To make a complete survey of the parent population is sometimes impossible or impractical. For example, it is impossible to measure the heights of all adult persons in a country. It is impractical to test for infectious bacteria the whole body of water in a city reservoir. All that we can do is to obtain an unbiased sample. By an unbiased sample, we mean a sample in which each individual has an equal and independent chance to be included. From this chosen sample we attempt to draw some conclusion concerning the nature of the whole parent population in accordance with certain mathematical principles.

Now the sample which we choose is of course only one of the samples that can be possibly drawn from a given parent population. Suppose there is a population of s individuals from which we wish to choose a sample of r. It is clear that there exist ${}_sC_r$ such samples, each of which is equally likely to be chosen. Therefore these ${}_sC_r$ samples constitute the so-called distribution of samples. To describe from the statistical point of view the distribution of samples, we must find its mean, standard deviation, skewness, excess, and other higher characteristics. The first three are usually referred to as elementary statistical functions.

Suppose x_i be the variate (by which we mean the magnitude of a specified character of an individual to be measured) where $i = 1, 2, 3, \cdots s$; and z_j be the samples chosen from the parent population where $j = 1, 2, 3, \cdots {}_sC_r$. Then the ${}_sC_r$ samples, each consisting of r variables, will be formed after the following fashion:

$$z_1 = x_1 + x_2 + x_3 + \cdots + x_r$$

$$z_2 = x_2 + x_3 + x_4 + \cdots + x_{r+1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$z_{\binom{s}{r}} = x_{s-r+1} + x_{s-r+2} + x_{s-r+3} + \cdots + x_s$$

---

[1] A dissertation submitted in partial fulfillment of the requirement for the degree of doctor of philosophy in the University of Michigan.

[2] The writer wishes to express his appreciation for the assistance Professor H. C. Carver has given him in making this study.