ON THE FREQUENCY DISTRIBUTION OF CERTAIN RATIOS

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Considerable interest in the distribution of ratios, $t = y/x$, has no doubt been suggested by important applications. For example, we may mention the opsonic index in bacteriology, the ratio of systolic to diastolic blood pressure in physiology, and ratios such as link relatives or certain index numbers in economics.

In 1910, Karl Pearson[1] gave certain properties of the distribution of ratios by means of approximate formulas for moments up to order four in terms of means, variances, product moments, and coefficients of variability of x and y. The resulting formulas did not give, with sufficient accuracy, the constants of the distribution of the opsonic index for the purpose of Dr. Greenwood to whom Pearson attributed the derivation of the formulas for the special case in which x and y are uncorrelated. Pearson next adopted the plan of tabulating the reciprocals, say $x' = \frac{1}{x}$, and then finding the constants of the distribution of the product yx' in the case in which x' and y are uncorrelated. He then obtained satisfactory results in illustrative examples.

In 1929, C. C. Craig[2] obtained the semi-invariants of y/x in terms of moments of x and y, and then expressed the moments in terms of the semi-invariants of the distribution function, $f(x, y)$, of x and y. By this means, he was able to deal with the case in which x and y are normally correlated under suitable conditions. Craig found it desirable to restrict the distribution of x in such a way that the probability of a zero value of x is an infinitesimal of sufficiently high order that a certain integral exists. This limitation seems to imply in applications to actual data that no zero values of x are to occur. This suggests that we deal with the cases of x at or near zero with considerable care.

By starting with the assumption that the values of x and y are a set of normally distributed pairs of values with correlation coefficient r, and by considering the quotient $z = \frac{b + y}{a + x}$, a and b being constants, R. C. Geary,[3] in a paper published in 1930, found an algebraic function, $u = f(z)$, of fairly simple form with the property that u is nearly normally distributed with arithmetic mean zero and standard deviation unity provided that $a + x$ is unlikely to

[1] On the constants of index distributions, Biometrika, Vol. 7 (1910), pp. 531–546.

[2] The frequency function of y/x, Annals of Mathematics, Vol. 30 (1928–29), pp. 471–486.

[3] The frequency distribution of the quotient of two normal variates, J. Royal Statistical Society, Vol. XCIII (1930), pp. 442–7.