SOME NOTES ON EXPONENTIAL ANALYSIS

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M. E. J. Geuhry de Bray in his charming little book "Exponentials made Easy"[1] tells how to determine the constants in the equation,

$$y = A_1 \epsilon^{a_1 x} + A_2 \epsilon^{a_2 x} \tag{I}$$

so that the curve will pass through four points, with equidistant ordinates on an empirical curve. If (Fig. 1) y_0, y_1, y_2, and y_3 are the equidistant ordinates and δ is their common separation, y_0 being the y intercept of the curve, de Bray's formulas are:

$$a_1 = \frac{\log z_1}{\delta}, \qquad a_2 = \frac{\log z_2}{\delta} \tag{II}$$

where z_1 and z_2 are the roots of the quadratic equation

$$\begin{vmatrix} z^2 & z & 1 \\ y_3 & y_2 & y_1 \\ y_2 & y_1 & y_0 \end{vmatrix} = 0. \tag{III}$$

The coefficients A_1 and A_2 of the two exponential terms are obtained by solving the two simultaneous equations

$$\begin{aligned} A_1 + A_2 &= y_0 \\ A_1 z_1 + A_2 z_2 &= y_1 \end{aligned} \tag{IV}$$

In attempting to find suitable empirical equations for some "river rating curves"—graphs of discharge versus stage—the writer tried to make use of de Bray's procedure. The original intention was to use the above method to determine the constants, and then to correct these constants by the use of Least Squares, as done by J. W. T. Walsh[2] in an application of the method to a problem in radioactivity. It often happens that a series of plotted observations suggest a simple exponential function, but that when the observations are replotted on semi-logarithmic paper a straight line is not obtained. Often, as in the case of a good many river rating curves, the result may be described

[1] Macmillan & Co. Ltd., St. Martin's St., London W. C. 2.

[2] Proceedings Phys. Soc. London XXXII. This reference is given by de Bray in his book, "Exponentials made Easy."