A METHOD OF DETERMINING THE REGRESSION CURVE WHEN THE MARGINAL DISTRIBUTION IS OF THE NORMAL LOGARITHMIC TYPE

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In a paper[1] in this Journal Professor S. D. Wicksell gave the general outlines of a new method of calculating the regression lines. This problem was later on treated in detail by Dr. Walter Andersson.[2] His method was to develop the formulas for the regression lines into a series of orthogonal polynomials under the assumption that the marginal distribution of the independent variate belonged to certain mathematically defined distributions, and to determine the constants with the aid of the method of the least squares.

Among other cases he treated also the case where the marginal distribution was of the normal logarithmic type:

$$F(x) = \frac{\log e}{\sigma_l \sqrt{2\pi}\,(x - a)}\, e^{-\frac{1}{2}\left[\frac{\log (x-a) - l}{\sigma_l}\right]^2}. \tag{1}$$

But as his method is entirely different from the method I shall give here, I will not go any further into the method used by Dr. Andersson.

When the correlation surface $F(x, y)$ of the variates x and y is given and then of course also the marginal distribution of x, $F(x)$, it is known that the mean y_x of the dependent variate y in an infinitely small array with the value of x between x and $x + dx$ is given as a function of the independent variate x by the following formula (2)

$$y_x = \frac{\int yF(x, y)\, dy}{\int F(x, y)\, dy}. \tag{2}$$

In this formula the integrals are to be extended over the whole domain of the variation of y.

If now we make any transformation of x by introducing a new variate u, related to x by the formula $u = \psi(x)$, where we must suppose that u is a one-valued function of x and contrary, the distribution $f(u, y)$ of the variate u and y is given by the relation

$$f(u, y)\, du\, dy = F(x, y)\, dx\, dy \tag{3}$$

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[1] S. D. Wicksell. Remarks on Regression. Annals of Mathematical Statistics, 1930.

[2] Walter Andersson. Researches into the theory of Regression. Meddelande från Lunds Astronomiska Observatorium. Ser II. N:r 64.