# ON A METHOD FOR EVALUATING THE MOMENTS OF A BERNOULLI DISTRIBUTION[1]

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1. The moments (per unit frequency) of a frequency distribution have long been regarded as useful characteristics of the distribution. If we denote the moment about the arithmetic mean by $\mu$, we have for the Bernoulli distribution

$$\mu_s = \sum_{x=0}^{n} (\bar{x})^s f(x),$$

where $\bar{x} = x - np$ and $f(x) = \binom{n}{x} p^x q^{n-x}$.

To evaluate the $s$-th moment about the arithmetic mean has always been a laborious task. Karl Pearson[2] gave the $s$-th moment about the arithmetic mean as,

$$\mu_s = \left[\frac{d^s}{dx^s}[qe^{px} + pe^{-qx}]^n\right]_{x=0}, \tag{1}$$

which he said at that time was perhaps the easiest expression for obtaining these moment coefficients by successive differentiation. Romanovsky,[3] however, was able to develop the recursion formula,

$$\mu_{s+1} = pq\left[ns\mu_{s-1} + \frac{d\mu_s}{dp}\right], \tag{2}$$

for the moments about the mean. Another relation for these moments is

$$\mu_{s+1} = \sum_{i=0}^{s-1} \binom{s}{i} [npq\mu_i - p\mu_{i+1}]. \tag{3}$$

Recently Kirkham[4] gave the expressions for the first eight moments which, however, are not in a form well adapted for numerical calculation on a machine.

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[1] Presented to the American Mathematical Society, January 2, 1936.

[2] Karl Pearson, *Biometrika*, vol. 12 (1918–1919), footnote, p. 270. This expression is obtained from the moment-generating function. Obviously this method is exceedingly impractical for numerical calculations.

[3] V. Romanovsky, "Note on the moments of the binomial $(p + q)^n$ about its mean," *Biometrika*, vol. 15 (1923). Recently this expression was given a simple proof by A. T. Craig (*Bull. Amer. Math. Soc.*, vol. 40, pp. 262–264) and extended to the Poisson case.

[4] W. J. Kirkham, "Moments about the arithmetic mean of a binomial frequency distribution," *Annals of Mathematical Statistics*, vol. VI, pp. 96–101.