MOMENTS OF ANY RATIONAL INTEGRAL ISOBARIC SAMPLE MOMENT FUNCTION

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Introduction

The problem of moments of moments has been investigated by a number of authors. The assumption of an infinite universe (or that of a finite universe with replacements) permits the application of the "algebraic" method, the method of semi-invariants as introduced by Thiele (1) and developed by C. C. Craig (2) and the combinatorial analysis method introduced by R. A. Fisher (3) and used by N. St. Georgescu (4). A combinatorial analysis method has the particular advantage that it enables one to compute separate terms of a given formula.

The formulae for moments of moments have been simplified through the use of new moment functions. Thiele introduced the half-invariant (1) which resulted in considerable condensation. More recently Prof. R. A. Fisher (3) has introduced the sample function k whose expected value is a half invariant. The most compact formulization presented thus far is his formulation of the half invariants of the sample k_r in terms of the half invariants of the universe. This very compactness, however, makes it difficult to compare results with those expressed in the more conventional sample functions. Dr. Wishart has written a paper (7) in which he shows, among other things, how the Fisher results can be translated to the more conventional (Craig) results and vice versa, but such translation is in general no simple matter. It appears that the Fisher results are not immediately useful to the statistician who desires the formulae to be expressed in terms of the usual sample moment function. On the other hand the Fisher formulization is a remarkable discovery toward that harmony which must be naturally inherent in the field of moments of moments. Soper (6, 111) expressed the general situation when he wrote, "If the terrifying overgrowth of algebraic formulation accompanying this branch of statistical inquiry is destined to have a chief utility in induction and going back to causes, then perhaps Dr. Fisher's way of estimating a sample will prove to be most fertile, but if it is to be applied to problems of deduction, say to problems of successive eventuation such as propagation, then Mr. Craig's plain moments seem to have a firmer hold on the exigencies of time."

It would appear then that the Fisher formulae and the Craig formulae are both needed. Georgescu (4) showed a partial connection between them in applying to the m functions a combinatory analysis somewhat similar to that applied by R. A. Fisher to the k function. It is the purpose of the present