

MOMENT RECURRENCE RELATIONS FOR BINOMIAL, POISSON AND HYPERGEOMETRIC FREQUENCY DISTRIBUTIONS¹

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1. **Introduction.** This paper gives the development of recurrence relations for moments about the origin and mean of binomial, Poisson, and hypergeometric frequency distributions from the basis of the moment arrays defined by H. E. Soper.² This procedure has the advantage of expressing the moments in terms of coefficients which are alike for the three distributions and are derivable by a single process, thus providing a degree of formal coordination of the distributions. For both kinds of moments, the coefficients satisfy relatively simple recurrence relations, the use of which leads to recurrence relations for the moments, thus unifying the derivation of these relations for the three distributions. The relations derived in this way for the hypergeometric distribution are apparently new. Apparently new recurrence relations for certain auxiliary coefficients in the expression of the moments about the mean of binomial and Poisson distributions are also given.

This course of development involves repetition of a number of well-known results which is justified, it is hoped, by the unification obtained.³

¹ Presented to the American Mathematical Society, Sept. 3, 1936.

² *Frequency Arrays*, Cambridge, 1922.

³ The following bibliography is taken from a paper *On the Bernoulli Distribution*, Solomon Kullback, *Bull. Am. Math. Soc.*, **41**, 12, pp. 857-864, (Dec., 1935):

A. Fisher, *The Mathematical Theory of Probabilities*, 2d ed., p. 104 ff.

H. L. Rietz, *Mathematical Statistics*, 1927, p. 26 ff.

V. Mises, *Wahrscheinlichkeitsrechnung*, 1931, pp. 131-133.

Risser and Traynard, *Les Principes de la Statistique Mathématique*, 1933, pp. 39-40 and 320-321.

V. Romanovsky, *Note on the moments of the binomial $(q + p)^n$ about its mean*, *Biometrika*, vol. 15 (1923), pp. 410-412.

A. T. Craig, *Note on the moments of a Bernoulli distribution*, *Bull. Am. Math. Soc.*, vol. 40 (1934), pp. 262-264.

A. R. Crathorne, *Moments de la binomiale par rapport à L'origine*, *Comptes Rendus*, vol. 198 (1934), p. 1202;

A. A. K. Aygangar, *Note on the recurrence formulae for the moments of the point binomial*, *Biometrika*, vol. 26 (1934), pp. 262-264.

To this, besides Soper's tract already mentioned, should be added:

Ch. Jordan, *Statistique Mathématique*, Paris, 1927.

K. Pearson, *On Certain Properties of the Hypergeometric Series . . .*, *Phil. Mag.*, **47**, pp. 236-246 (1899).