DISTRIBUTIONS OF SUMS OF SQUARES OF RANK DIFFERENCES
FOR SMALL NUMBERS OF INDIVIDUALS

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I. INTRODUCTION

In a recent article,\(^2\) reporting the results of research under a grant-in-aid from
the Carnegie Corporation of New York, Hotelling and Pabst have given a
comprehensive treatment of the theory and application of rank correlation and
have contributed significantly to existing knowledge on the subject. It is not
the purpose of this note to evaluate their contribution but to attempt the
solution of a problem they suggest.

In §3\(^3\) they have given the well-known formula for rank correlation, \(r' = 1 - \frac{6\Sigma d^2}{n(n-1)}\), where \(n\) is the number of individuals ranked and \(\Sigma d^2 = \sum_{i=1}^{n} d_i^2\) (\(d_i\) being
the rank difference for the \(i\)th individual). In §5 the question of the significance
of \(r'\) in small samples has been considered from the following point of view; if the
value of \(r'\), obtained from a comparison of the ranks of \(n\) individuals as a possible
measure of the relation between two attributes, is such that there exists a high
probability that it could have occurred by virtue of a chance rearrangement of
the \(n\) individuals, then the value of \(r'\) does not furnish a significant indication of
relationship. Then one test of the significance of a particular value of \(r'\) is to
note whether it has a probability less than \(P\) (\(P\) equal to .01 or, less stringently,
equal to .05) of occurring because of a chance re-ranking.

To apply this test it is necessary to have some information regarding the
distribution of \(r'\) for the chance rearrangements of the numbers from 1 to \(n\).
Hotelling and Pabst have given the distribution of \(r'\) for the cases, \(n = 2, 3, 4\).
They have noted that the distribution is symmetrical for each value of \(n\) and
that it has a range from \(-1\) to 1. From a consideration of the probabilities

\(^1\) Presented to the American Mathematical Society, December 29, 1936.
\(^2\) Harold Hotelling and Margaret Pabst, Rank Correlation and Tests of Significance
1936, pp. 29-43.
\(^3\) Loc. cit.

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