SOME EFFICIENT MEASURES OF RELATIVE DISPERSION[1]

By Nilan Norris

For some time it has been known that the coefficient of variation (in the sense of the ratio of the standard deviation to the arithmetic mean) is not an efficient statistic for distributions departing materially from normality.[2] At various times there have been proposed certain supplementary estimates of relative variation, such as those involving ratios between sums and differences of upper and lower quartiles, and ratios of mean deviations to medians or to arithmetic means. Some of these have appeared in certain textbooks.[3] But there appears to have been no attempt to found their use on considerations of minimum sampling variance.

The point of departure of this paper is that of using the Method of Maximum Likelihood to derive two efficient measures of relative dispersion, together with expressions for their standard errors. These optimum estimates of true or parametric variation are the ratio of the arithmetic mean to the geometric mean (the arithmetic-geometric ratio) for Pearson Type III distributions, and the ratio of the geometric mean to the harmonic mean (the geometric-harmonic ratio) for Pearson Type V distributions. The usefulness of these measures is suggested by the generalized-mean-value-function approach to the analysis of averages, especially the theorem of inequalities among averages.[4]

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[2] The term "efficient statistic" is used here in the sense of R. A. Fisher, that is, of a parameter-estimate which tends towards normality of distribution with the least possible standard deviation. For a discussion of the inefficiency of certain commonly used statistics as applied to distributions departing from normality, see R. A. Fisher, "On the Mathematical Foundations of Theoretical Statistics," *Philosophical Transactions of the Royal Society of London*, Series A, Vol. 222, 1922, pp. 332–336.

[3] See, for example, William Vernon Lovitt and Henry F. Holtzclaw, *Statistics* (Prentice-Hall, Inc., New York, 1929), p. 134; Herbert Arkin and Raymond R. Colton, *Statistical Methods* (Barnes and Noble, Inc., New York, 1935), revised ed., p. 41; and Herbert Sorenson, *Statistics for Students in Psychology and Education* (McGraw-Hill Book Company, Inc., New York, 1936), pp. 153 *f.*

[4] Nilan Norris, "Inequalities among Averages," *Annals of Mathematical Statistics*, Vol. VI, No. 1, March, 1935, pp. 27–29; and "Convexity Properties of Generalized Mean Value Functions," *Annals of Mathematical Statistics*, Vol. VIII, No. 2, June, 1937, pp. 118–120. Professor John B. Canning appears to have been the first to point out the possibility of making use of certain ratio measures of relative variation. See "The Income Concept and Certain of Its Applications," *Papers and Proceedings of the Eleventh Annual Conference of the Pacific Coast Economic Association* (Edwards Brothers, Ann Arbor, 1933), p. 64.