A TEST OF THE SIGNIFICANCE OF THE DIFFERENCE BETWEEN MEANS OF SAMPLES FROM TWO NORMAL POPULATIONS WITHOUT ASSUMING EQUAL VARIANCES[1]

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1. **History of the problem.** If the only available evidence about two normally distributed populations is contained in two samples, one from each, it has hitherto been the custom to the test the hypothesis that the means are equal by assuming that the quantity $\frac{\bar{x} - \bar{x}'}{\sqrt{ks^2 + k's'^2}}$ is distributed in Student's distribution, with $N + N' - 2$ degrees of freedom, where $s^2 = \frac{\Sigma(x - \bar{x})^2}{N(N-1)}$ and $k = \frac{(N-1)(N+N')}{N'(N+N'-2)}$, the other notation being that used by R. A. Fisher.[2] The hypothesis underlying this test, however, is that the variances are equal. Although in many cases this may seem a reasonable assumption to adopt concurrently with that of equal means, it is undoubtedly not a necessary one, and it is, therefore, desirable that the test should be adapted to meet this difficulty.

The first advance on the problem was made by W. V. Behrens[3] who suggested that the distribution of the difference of the means could be expressed in terms of the observations in the samples from the two populations, the argument being entirely independent of the variances. R. A. Fisher[4] obtained substantially the same result, but expressed the argument in terms of fiducial probability. M. S. Bartlett[5] was of the opinion that Behrens' reasoning was incorrect, as he obtained some results which were apparently inconsistent with those tabulated in Behrens' paper, but R. A. Fisher[6] showed that Bartlett's argument was open to criticism. In the latter work, he actually obtained distributions for the case of two samples of two observations, and in the following we shall indicate some extensions of this more detailed work of Fisher, firstly, to the case

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[1] Presented at the joint meeting of the American Mathematical Society and the Institute of Mathematical Statistics, Indianapolis, December 30, 1937.

Research done under a grant-in-aid from the Carnegie Corporation of New York City.

[2] *Statistical Methods for Research Workers*, 1925–1936.

[3] "Ein Beitrag zur Fehlerberechnung bei wenige Beobachtungen," *Landw. Jb.* 68, 807–37 (1929).

[4] "The Fiducial Argument in Statistical Inference," *Annals of Eugenics*, 6 (1935) pp. 391–8.

[5] "The Information Available in Small Samples," *Proc. Camb. Phil. Soc.* 32, pp. 560–6 (1936).

[6] "On a Point Raised by M. S. Bartlett on Fiducial Probability," *Annals of Eugenics* 7 Part IV, 370–5 (1937).