TRANSFORMATIONS OF THE PEARSON TYPE III DISTRIBUTION

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I. Introductory

Transformations of the normal curve have been used as a basis for the representation of skew frequency distributions by Edgeworth, Kapteyn, Van Uven, Bernstein, and others. Various studies have been made of the distributions obtained by replacing each of a set of normally distributed variates by a logarithmic function of the variates. Among the earlier investigators along this line were Galton, and McAllister; later, works by Jorgensen, Fisher, Wicksell, Davies, and a more recent study by Pae-Tsi-Yuan, were added.

Rietz[1] restated and treated, in a general fashion, the question as to the properties of the distribution of powers of a set of variates which are known to be normally distributed. By a suitable choice for the origin of the normal curve, he obtained results which are applicable in answering questions which frequently arise in the applied field concerning the properties of families of interrelated distributions, one strain of which is known to be normally distributed. For example, in the family made up of the diameters, surface areas, volumes, etc. of some physical quantity, if it were known that one set, the surface areas for instance, were distributed normally, then from his results we have the properties of the distributions of any of the other sets.

Likewise it has seemed of interest to investigate, in a similar fashion, the properties of the transformed Type III Pearson distribution. We shall treat both the power and logarithmic transformations. For instance, if we knew that any one of the physical measurements, velocity, kinetic energy, momentum, or centrifugal force (all of which are functions of the velocity) were distributed according to a Type III curve, then we raise the question as to the properties of the distributions of any of the others. Similarly, if the intensity of certain light, I, were known to be distributed according to a Type III law, we will discuss the properties of the distribution of the brightness, B, of the light as seen by the eye, since the two are known to be related by the law $B = K \log I$. The same analysis applies to the relationship between L, the loudness of a sound, and E, the energy in the sound wave, since $L = K \log E$.

Two forms of the Type III distribution will be considered. In the first form, all the variates are taken positive; in the second form, the origin is at the mean and the variates are measured in units of standard deviation.

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[1] H. L. Rietz, Frequency Distributions Obtained By Certain Transformations of Normally Distributed Variates, Annals of Math., Vol. 23, (1922) pp. 291–300.