SHORTEST AVERAGE CONFIDENCE INTERVALS FROM LARGE SAMPLES

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1. **Introduction.** The method of fiducial argument [1, 2] in statistics has gained considerable prominence within the last few years as a method of inferring the values of population parameters from samples "randomly drawn" from populations having distribution laws of known functional forms. The method has also been shown to be applicable [2] to the problem of inferring the values of statistical functions in samples from samples already observed, assuming all samples to be drawn from a population with a distribution law of a given functional form.

The main ideas of a procedure which is sufficient for carrying out fiducial inference for certain cases of a single population parameter may be summed up in the following steps:

(a) A sample is assumed to be "randomly drawn" from a population with a distribution law $f(x, \theta)$ of known functional form.

(b) A function $\psi(x_1, x_2, \cdots, x_n, \theta)$ of the sample values $x_1, x_2, \cdots, x_n$ and parameter θ is devised, which is a monotonic function of θ for a given sample, so that the sampling distribution $G(\psi)d\psi$ of $\psi(x_1, x_2, \cdots, x_n, \theta_0) = \psi_0$, say, in samples from the population with $\theta = \theta_0$ is independent of θ_0 and the x's, except as they enter into ψ.

(c) For a given probability α a pair of numbers ψ'_α and ψ''_α is chosen so that when $\theta = \theta_0$, the probability that $\psi'_\alpha < \psi_0 < \psi''_\alpha$ is $1 - \alpha$, or more, briefly,

$$P(\psi'_\alpha < \psi_0 < \psi''_\alpha \mid \theta = \theta_0) = 1 - \alpha \tag{1}$$

which can be stated in the alternative form

$$P(\underline{\theta} < \theta_0 < \bar{\theta} \mid \theta = \theta_0) = 1 - \alpha. \tag{2}$$

(d) $\underline{\theta}$ and $\bar{\theta}$ being functions of ψ'_α, ψ''_α and the sample, are subject to sampling fluctuations and it can be stated that the probability is $1 - \alpha$ that they will include the true value of θ, whatever it may be, that is, θ_0, between them. The statement holds for all values which θ_0 may take on.

The numbers $\underline{\theta}$ and $\bar{\theta}$ are known as *fiducial* or *confidence limits* [3] of θ_0 and $(\underline{\theta}, \bar{\theta})$ a *confidence interval* for the *confidence coefficient* $1 - \alpha$. We therefore have the following rule for making inferences about the unknown number θ_0 once ψ has been found: For a given sample solve the equations

$$\psi(x_1, x_2, \cdots, x_n, \theta_0) = \psi'_\alpha, \qquad \psi(x_1, x_2, \cdots, x_n, \theta_0) = \psi''_\alpha$$