INTERIOR AND EXTERIOR MEANS OBTAINED BY THE METHOD OF MOMENTS

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1. Introduction—The Substitutive Mean. A very general mean based upon substitution was proposed by O. Chisini.[1] Briefly stated, this mean M of data x_1, x_2, $\cdots$, x_n, is a number which satisfies some equation of the form

$$(1) \qquad G(M, M, \cdots, M) = G(x_1, x_2, \cdots, x_n).$$

If, now,[2]

$$(2) \qquad M = F(x_1, x_2, \cdots, x_n)$$

is an *explicit* expression of M, then for each value c which each of the arguments x_i can take on,

$$(3) \qquad F(c, c, \cdots, c) = c;$$

or at least one value of this F is c.

Suppose now that $F(x_1, x_2, \cdots, x_n)$ is *any* function of x_1, x_2, $\cdots$, x_n, defined for at least one set of equal arguments c, and such that whenever defined for equal arguments c, at least one value of $F(c, c, \cdots, c) = c$. Such a function, I have called a *substitutive mean*. Various extensions[3] are immediate, such as the use of integration in place of summation. Indeed, point set functions or functionals may be used.[4] Here I shall supplement (3) by a fairly common convention. If $F(c, c, \cdots, c)$ is not originally defined, but as $x_i \to c$ simultaneously, limit $F(x_1, x_2, \cdots, x_n) = c$, in this case, $F(c, c, \cdots, c)$ will be assigned its limiting value c,—thus establishing continuity.

2. Location and scale as means. The purpose of this paper is to investigate the nature of the means which arise when the well known *Method of Moments* is used to estimate the values of two important parameters—namely, the location κ and the scale α of a frequency function or distribution. These are taken as associated with the variable x of the distribution thus:

$$(4) \qquad t' = (x - \kappa)/\alpha.$$

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[1] O. Chisini, "Sul Concetto di media," *Periodico di Matematico*, Series 4, Vol. 9, (1929), pp. 106–116.

[2] E. L. Dodd, "Internal and External Means Arising from the Scaling of Frequency Functions," These *Annals*, Vol. 8, (1937), pp. 18–20.

[3] For an extension of Chisini's results, see: Bruno de Finetti, "Sul Concetto di media," *Giornale dell' Instituto Italiano degli Attuari*, Vol. 2, (1931), pp. 369–396.

[4] E. L. Dodd, "The Chief Characteristic of Statistical Means," Cowles Commission lecture. *Colorado College Publication*, General Series No. 208, (1936), pp. 89–92.