ABSTRACTS OF PAPERS

(Presented on December 27, 1938, at the Detroit meeting of the Institute)

Generalizations of the Laplace-Liapounoff Theorem. W. G. MADOW, Milbank Management Corporation, New York.

The Laplace-Liapounoff Theorem states conditions under which a linear function of chance variables has a normal limiting distribution.

In dealing with limiting distributions arising in the analysis of variance, regression analysis, etc., there occurred problems which required for their solution the derivation of the joint limiting distribution of several linear functions of chance variables and the joint limiting distribution of functions which were linear in one set of chance variables for fixed values of other sets of chance variables.

These problems were solved by a matrix formulation of the Laplace-Liapounoff Theorem and by the introduction of a function whose convergence to zero in probability provided a sufficient condition for the existence of normal limiting distributions.

Various generalizations with a view towards applications in multi-variate statistical analyses are discussed. The theorems provide a rigorous and complete basis for the derivation of limiting distributions of quadratic and bilinear forms.

The Standard Errors of the Geometric and Harmonic Means. NILAN NORRIS, Hunter College.

Although certain properties of the geometric and harmonic means have been investigated extensively, there seems to have been no derivation of expressions for their variances in cases where they are used as estimates of parameters of parent populations.

Application of the modern theory of estimation makes it possible to develop simple and useful formulae for the standard errors of these two averages for each of the respective general classes of cases in which they are most suitable.

As in other instances in which standard errors are used in tests of significance, fiducial or confidence limits may be employed to overcome certain limitations of the outmoded practice of relying solely on multiples of either probable or standard errors to determine whether or not a result exists merely because of sampling fluctuations.

Note on an Integral Equation in Population Analysis. ALFRED J. LOTKA, Metropolitan Life Insurance Company, New York.

In a population in which immigration and emigration are negligible, the number $N(t)$ of the population at time t is connected with the annual births $B(t)$ and the probability $p(a)$ of surviving from birth to age a, by the obvious relation

$$N(t) = \int_0^\infty B(t - a)p(a)\, da. \tag{1}$$

If $B(t)$ and $p(a)$ are given, $N(t)$ follows at once by direct integration. The inverse problem, given $N(t)$, to find $B(t)$, requires separate treatment. The case that $N(t)$ is given or can be expressed as a sum of exponential functions has been discussed by the