ON CONFIDENCE LIMITS AND SUFFICIENCY, WITH PARTICULAR REFERENCE TO PARAMETERS OF LOCATION

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1. **Introduction.** The solution of the problem of estimating an interval in which a population parameter should lie, by means of what is now often termed the fiducial type of argument, dates back to the early writers on the theory of errors. However, owing to their lack of "Student's" z distribution, their statements were usually only of an approximate character, and, furthermore, the logical distinction between the fiducial method and the method of inverse probability was never clearly drawn, before R. A. Fisher discussed the subject. It is of interest to note how far "Student" himself went in this matter. In describing the tables which he gave in his original paper he says:[1]

"The tables give the probability that the value of the mean, measured from the mean of the population, in terms of the standard deviation of the sample, will lie between $-\infty$ and z. Thus, to take the tables for samples of six, *the probability of the mean of the population lying between* $-\infty$ *and once the standard deviation of the sample is* 0.9622 or the odds are about 24 to 1 that the mean of the population lies between these limits. The probability is therefore 0.0378 that it is greater than once the standard deviation, and 0.0756 that it lies outside $\pm$ 1.0 times the standard deviation."

It should be noted that "Student's" z is $(\bar{x} - \theta)/s$ where θ is the true population mean. His tables tell us that for $n = 6$, $P(z < 1)$[2] is equal to 0.9622. Owing to the symmetry of the z distribution this is equivalent to saying that $P(z > -1)$ is 0.9622, i.e.

$$P\left\{\frac{\bar{x} - \theta}{s} > -1\right\} = 0.9622.$$

This may be transposed to read

$$P\{\theta < \bar{x} + s\} = 0.9622 \tag{1}$$

which is the statement I have italicized in the above extract, it being there understood that the mean of the population is being measured from the mean of the sample. "Student" therefore makes here what is now called a fiducial statement. In the next sentence he, in effect, attaches a probability to an interval estimate for the population mean. In doing this "Student" was not conscious of introducing any new principle, nor does he apply the method consistently

[1] "Student" (1908). "The Probable Error of a Mean." *Biometrika* VI, p. 20.

[2] P is used to denote the probability of the truth of the relation in the bracket following.