NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

NOTE ON THE L_1 TEST FOR MANY SAMPLES

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Neyman and Pearson[1] have discussed a method for testing the hypothesis that k samples have been drawn from normal populations with the same variances by means of a statistical function, L_1, defined by

$$L_1^{\frac{N}{2}} = \prod_{t=1}^{k} \left(\frac{s_t^2}{s^2}\right)^{\frac{n_t}{2}}$$

where n_t is the number of elements in the t-th sample, s_t^2 is the sample variance and

$$s^2 = \sum_{t=1}^{k} \frac{n_t}{N} s_t^2 \qquad N = \sum_{t=1}^{k} n_t .$$

For convenience, we shall denote $L_1^{\frac{N}{2}}$ by λ. In their paper Neyman and Pearson have found the moments of λ and have shown that the distribution of $-2 \log_e \lambda$ approaches that of χ^2 with $k - 1$ degrees of freedom when the number of elements in each of the k samples becomes large. In some applications of this test the question arises as to whether the χ^2 law is a good approximation when the number of samples is large in comparison with the number of elements in each sample. For example, in a certain educational study, the number of schools was much greater than the number of pupils in each school, and it was desired to test for heterogeneity of variances of scores on a given examination using L_1 as the criterion. The purpose of this note is to examine the behavior of the L_1 test for large values of k.

Wilks has obtained the distribution of λ as a definite integral; it is, however, a rather cumbersome form to handle. The procedure here will be simply to compare the first few semi-invariants of $-2 \log \lambda$ with those of χ^2. The p-th moment of λ is[2]

$$(1) \qquad \mu'(p) = \frac{N^{\frac{pN}{2}} \Gamma\left(\frac{N-k}{2}\right)}{\Gamma\left(\frac{(p+1)N-k}{2}\right)} \prod_{t=1}^{k} \frac{\Gamma\left(\frac{(p+1)n_t - 1}{2}\right)}{n_t^{\frac{pn_t}{2}} \Gamma\left(\frac{n_t - 1}{2}\right)}.$$

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[1] "On the problem of k Samples," *Bulletin de l'Académie Polonaise des Sciences et des Lettres*, Série A (1931), pp. 460–481.

[2] Ibid., p. 472.