COMPLETE SIMULTANEOUS FIDUCIAL DISTRIBUTIONS

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1. Introduction. In a recent paper in these Annals, Starkey [13] has made some investigation of the distribution[1] related to the Behrens-Fisher test of the difference between two means from normal populations with unequal variances. She does not, however, give any critical discussion of the validity of this proposed test in the light of criticisms that have been made of it. It may therefore be an appropriate opportunity of reviewing the theory of fiducial distributions, as I see it, up to the present stage of development,[2] and in particular, of referring to the idea of complete simultaneous fiducial distributions. In conclusion I have made some brief comment on the particular problem at issue, in the light of this general theory; and have added a note on the use of approximate tests.

2. Fiducial Probability. If from a sample denoted symbolically by S a statistic T is obtained whose chance distribution depends on one unknown parameter θ, the distribution of T being of the form

$$p(T \mid \theta) = f(T, \theta)\, dT,$$

and if the values of T bear a regular increasing relationship with θ, (for an assigned value of the probability integral), then for any particular value $T = T_0$, we may assert that $\theta \geq \theta_0$, where

$$\int_{-\infty}^{T_0} p(T \mid \theta_0) = 1 - \epsilon,$$

and we shall know that this assertion, in the system of inferences based on the above rule, will have an exact and known probability of being wrong, given by ϵ.

The inference is thus an uncertain one, but the extent of the uncertainty is exactly known, and as stressed by Fisher [6], who first introduced this important concept of fiducial inferences and fiducial probability, is completely independent of any *a priori* notion of what value θ is likely to be.

It might be emphasized, to avoid confusion, that the inference is a *deduction* from the standpoint of logic, and still requires, if applied in practice, the necessity of inductive assumptions concerning the applicability of the mathematical theory, but its avoidance of any appeal to *a priori* probability in regard to the value of θ gives it a completely independent status district from the classical inverse probability argument, from which it should be distinguished. The

[1] This distribution has also been studied by Sukhatme [14].
[2] See also the recent expository article by Wilks [16].