

**ON THE PROBABILITY OF ATTAINING A GIVEN STANDARD DEVIATION RATIO IN AN INFINITE SERIES OF TRIALS**

BY JOSEPH A. GREENWOOD AND T. N. E. GREVILLE

Suppose an event with constant probability  $p$  of occurrence to be repeated an infinite number of times, and suppose the ratio of the deviation from the expected number of successes to the standard deviation  $\sqrt{npq}$  to be recomputed after each trial. We are interested in the probability that this ratio will at some time equal or exceed some positive number  $k$ . It is not difficult to show that the value of this probability is unity, but as the fact has not, to our knowledge, been previously pointed out in the literature, we give the following proof.

Let  $x_n$  denote the number of successes obtained in the first  $n$  trials, let

$$t_n = \frac{x_n - np}{\sqrt{npq}},$$

and let  $P$  denote the probability that, for some  $n$ ,  $t_n \geq k$ . We shall prove that  $P = 1$ . To do this, let the infinite series of trials be subdivided into consecutive, mutually exclusive subseries of finite length, and let  $m_i$  denote the number of trials in the  $i$ -th subseries. Let  $N_i = \sum_{j=1}^{i-1} m_j$  for  $i \geq 2$ , while  $N_1 = 0$ . Let  $k'$  be any number greater than  $k$ , and let  $m_i$  be so chosen that

$$(1) \quad m_i \geq \frac{k'^2 p}{q} \quad \text{for every } i,$$

and

$$(2) \quad \sqrt{m_i} \left( k' - k \sqrt{\frac{N_i + 1}{m_i}} \right) \geq N_i \sqrt{\frac{p}{q}} \quad \text{for } i \geq 2.$$

It follows from (1) that

$$(3) \quad m_i \geq m_i p + k' \sqrt{m_i p q} \quad \text{for every } i.$$

It follows from (2) that

$$(4) \quad m_i p + k' \sqrt{m_i p q} \geq (N_i + m_i) p + k \sqrt{(N_i + m_i) p q} \quad \text{for every } i.$$

Let  $y_i$  denote the number of successes in the  $i$ -th subseries. It is evident from (4) that if

$$(5) \quad y_i \geq m_i p + k' \sqrt{m_i p q}$$

for any  $i$ , then

$$t_{N_i + m_i} \geq k.$$

Hence  $P$  is at least equal to the probability that (5) holds for some  $i$ .

Let  $p_i$  denote the probability that (5) holds for a particular  $i$ . It follows from (3) that, for every  $i$ ,  $p_i > 0$ . Moreover, there exists a positive integer  $M$

