THE PROBLEM OF m RANKINGS

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1. **Introduction.** If n objects are ranked by m persons according to some quality of the objects there arises the problem: does the set of m rankings of n show any evidence of community of judgment among the m individuals? For example, if a number of pieces of poetry are ranked by students in order of preference, do the rankings support the supposition that the students have poetical tastes in common, and if so is there any strong degree of unanimity or only a faint degree?

The problem in its full generality permits of no assumption about the nature of the quality according to which the objects are ranked, other than that ranking is possible. No hypothesis is made that the quality is measurable, still less that there is some underlying frequency distribution to the quantiles of which the rankings correspond. The quality is to be thought of as linear in the sense that any two objects possessing it are either coincident or may be put in the relation "before and after." A metric may, of course, be imposed on this linear space by convention; but the relationship between objects is invariant under any transformation which stretches the scale of measurement. In particular, it is not a condition of the problem that the ranking shall be based on a distribution according to a normal variate.

It is permissible to denote the rankings by the *ordinal* numbers 1, 2, $\cdots n$; but it is not permissible, without further discussion, to operate on these numbers as if they were cardinals. This point seems to have been inadequately appreciated. For instance, when $m = 2$ we have the familiar case of rank correlation between a pair of rankings; and this is frequently treated by subtracting corresponding ranks, squaring, and forming the Spearman coefficient

$$\rho = 1 - \frac{6S(d^2)}{n^3 - n}. \tag{1}$$

To justify this procedure it is necessary to explain what is meant, for example, by such a process as (4th minus 8th), or what the square of this difference of ordinal numbers represents.

It is worth stressing that the necessary transition from ordinals to cardinals can be made without invoking a scale of measurement. When we rank an object as first we mean, in effect, that no member of the set of n is preferred to it; when we rank it as the rth we mean that $(r - 1)$ objects are preferred to it. The ordinals of the ranking are then biunivocally related to the cardinals expressing the number of objects which are preferred. It is thus legitimate