CONTRIBUTIONS TO THE THEORY OF STATISTICAL ESTIMATION AND TESTING HYPOTHESES[1]

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1. **Introduction.** Let us consider a family of systems of n variates $X_1(\theta^{(1)}, \cdots, \theta^{(k)}), \cdots, X_n(\theta^{(1)}, \cdots, \theta^{(k)})$ depending on k parameters $\theta^{(1)}, \cdots, \theta^{(k)}$. A system of k values $\theta^{(1)}, \cdots, \theta^{(k)}$ can be represented in the k-dimensional parameter space by the point θ with the co-ordinates $\theta^{(1)}, \cdots, \theta^{(k)}$. Denote by Ω the set of all possible points θ. For any point θ of Ω we shall denote by $P(E \epsilon w|\theta)$ the probability that the sample point $E = (x_1, \cdots, x_n)$ falls into the region w of the n-dimensional sample space, where x_j denotes the observed value of the variate $X_j(\theta)(j = 1, \cdots, n)$. The distribution $P(E \epsilon w|\theta)$ is supposed to be known for any point θ of Ω. In the theory of testing hypotheses and of statistical estimation we have to deal with problems of the following type: A sample point $E = (x_1, \cdots, x_n)$ of the n-dimensional sample space is given. We know that x_j is the observed value of $X_j(\theta)$ but we do not know the parameter point θ, and we have to draw inferences about θ by means of the sample point observed. The assumption that θ belongs to a certain subset ω of Ω is called a hypothesis. We shall deal in this paper with the following general problem: Let us consider a system S of subsets of Ω. Denote by H_ω the hypothesis corresponding to the element ω of S, and by H_S the system of all hypotheses corresponding to all elements of S. We have to decide by means of the observed sample point E which hypothesis of the system H_S should be accepted. That is to say for each H_ω we have to determine a region of acceptance M_ω in the n-dimensional sample space. The hypothesis H_ω will be accepted if and only if the sample point E falls in the region M_ω. M_ω and $M_{\omega'}$ are disjoint if $\omega \neq \omega'$. The statistical problem is the question as to how the system M_S of all regions M_ω should be chosen.

The problem in this formulation is very general. It contains the problems of testing hypotheses and of statistical estimation treated in the literature.[2] For instance if we want to test the hypothesis H_ω corresponding to a certain subset ω of Ω, the system of hypotheses H_S consists only of the two hypotheses H_ω and $H_{\bar{\omega}}$ where $\bar{\omega}$ denotes the subset of Ω complementary to ω. If we want to estimate θ by a unique point, then S is the system of all points of Ω. In the theory of confidence intervals we estimate one of the parameter co-ordinates $\theta^{(1)}, \cdots, \theta^{(k)}$,

[1] Research under a grant-in-aid from the Carnegie Corporation of New York.

[2] See, for instance, J. Neyman, "Outline of a Theory of Statistical Estimation Based on the Classical Theory of Probability," *Phil. Transactions of the Royal Society*, London, Vol. 231 (1937), pp. 333–380.