tion $\theta_i = \theta_i(\theta_1', \cdots, \theta_l')$ $(i = 1, \cdots, l)$ leads to the equally valid representation of the family

$$p'(x_1, \cdots, x_n \mid \theta_1', \cdots, \theta_l') = p[x_1, \cdots, x_n \mid \theta_1(\theta_1', \cdots, \theta_l'), \cdots, \theta_l(\theta_1', \cdots, \theta_l')].$$

Is a set of statistics sufficient with respect to the first representation also sufficient with respect to the second? The answer is partly in the affirmative and is given by the following proposition.

THEOREM II. *If the set of algebraically independent statistics $T_1, \cdots, T_m$ is sufficient with regard to the parameters $\theta_1, \cdots, \theta_q$ and the probability law $p(x_1, \cdots, x_n \mid \theta_1, \cdots, \theta_q, \cdots, \theta_l)$, it is also sufficient with regard to $\theta_1', \cdots, \theta_q'$ and any other representation $p'(x_1, \cdots, x_n \mid \theta_1', \cdots, \theta_q', \cdots, \theta_l')$ of the same probability law provided θ_i' $(i = 1, \cdots, q)$ are independent functions of $\theta_1, \cdots, \theta_q$ only and θ_j' $(j = q + 1, \cdots, l)$ are functions of $\theta_{q+1}, \cdots, \theta_l$ only.*

PROOF: The proof of the theorem is obvious. We are given the fact that $p(x_1, \cdots, x_n \mid \theta_1, \cdots, \theta_q, \cdots, \theta_l) = p(T_1, \cdots, T_m \mid \theta_1, \cdots, \theta_q)\cdot\phi(x_1, \cdots, x_n; \theta_{q+1}, \cdots, \theta_l)$. Since the θ_i' $(i = 1, \cdots, q)$ are functions of $\theta_1, \cdots, \theta_q$ only and the θ_j' $(j = q + 1, \cdots, l)$ are functions of $\theta_{q+1}, \cdots, \theta_l$ only, it follows that $\theta_i = \theta_i(\theta_1', \cdots, \theta_q')$ $(i = 1, \cdots, q)$ and $\theta_j = \theta_j(\theta_{q+1}', \cdots, \theta_l')$ $(j = q + 1, \cdots, l)$. Consequently,

$$(4) \quad p'(x_1, \cdots, x_n \mid \theta_1', \cdots, \theta_q', \cdots, \theta_l') = p'(T_1, \cdots, T_m \mid \theta_1', \cdots, \theta_q')\cdot\phi'(x_1, \cdots, x_n; \theta_{q+1}', \cdots, \theta_l')$$

and the theorem is established.

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NOTE ON THE MOMENTS OF A BINOMIALLY DISTRIBUTED VARIATE

BY W. D. EVANS

J. A. Joseph, has given two interesting triangular arrangements of numbers, the second of which is reproduced herewith as Table 1.[1] The successive rows in this table are the coefficients in the expansion of x^n as a function of the factorials $x^{(i)}$, using the notation of the calculus of finite differences. For example,

$$x^4 = x^{(4)} + 6x^{(3)} + 7x^{(2)} + x,$$

where

$$x^{(i)} = x(x - 1)(x - 2) \cdots (x - i + 1).$$

Joseph points out that the coefficients may be used to generate the numbers of Laplace.

[1] J. A. Joseph, "On the Coefficients of the Expansion of $X^{(n)}$," *Annals of Math. Stat.*, Vol. X (1939), p. 293.