

## ENUMERATION AND CONSTRUCTION OF BALANCED INCOMPLETE BLOCK CONFIGURATIONS<sup>1</sup>

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1. **Introduction.** One of the general problems of experimental design is to avoid extraneous effects in making desired comparisons. The method employed is to use experimental materials as nearly homogeneous as possible. Such materials, however, are seldom available in large quantities. On the contrary, field soils vary in fertility from block to block, animals vary with both litter and sex, and leaves on one young plant differ from those on another. Differences between blocks, between litters and sex, and between plants, being irrelevant to the comparisons usually contemplated, must be avoided.

When the number of treatments to be compared is small, well known methods of design, such as the Latin square or randomized complete block, are available and efficient. As the number of treatments increases, however, these designs tend to become less efficient through failure to eliminate heterogeneity. Furthermore, they become cumbersome, the Latin square design requiring replicates equal in number to the treatments and the complete block design providing that each treatment occur in every block. (Blocks are defined as an assemblage of experimental units chosen to be as nearly alike as possible.)

Because of such limitations, several modifications of the complete block design have been devised. These new designs all have the common characteristic that the experimental material is divided into groups or blocks containing fewer units than the number of treatments to be compared. These more homogeneous small blocks are referred to as incomplete blocks.

It is desirable to have all comparisons between pairs of treatments made with equal accuracy. This requires of the design that every pair of treatments occur in the same block an equal number of times. Such a design is referred to as balanced. Balanced incomplete block designs can be arranged (for any given number of treatments) only for certain combinations of block size and number of replications.<sup>2</sup>

The construction of balanced incomplete block designs is mathematically a part of the theory of configurations. A configuration is an assemblage of elements into sets, each element occurring in the same number of sets, and each

<sup>1</sup> A revision of an expository paper presented under a different title at a joint meeting of the Institute of Mathematical Statistics and Biometric Section of the American Statistical Association, December 27, 1939.

<sup>2</sup> Numerous additional designs are available in the partially balanced incomplete blocks [3].