

from which we can find

$$\rho_{V C^2} \sigma_V \sigma_{C^2} = \frac{2n(1-m)}{m(n+2)(m+n)}$$

and

$$\rho_{U C^2}^2 \cong \rho_{V C^2}^2 = \frac{(n+3)(n+4)(m+n-1)}{(n+1)(m+n+1)(m+n+2)}.$$

If  $n/m = \gamma$  (a fixed constant) and  $n$  is large

$$\rho^2 \cong \frac{n}{n+m}.$$

$\rho^2$  will be near 1 when  $n$  is much larger than  $m$ . This corresponds, in computing  $C^2$ , to dividing the smaller sample into subgroups by the larger. In this case  $U$  and  $C^2$  give essentially the same information. When  $m$  and  $n$  are more nearly equal the two criteria are quite different. For  $n > m$ ,  $C^2$  has fewer possible values than for  $n < m$ , and is therefore a more sensitive test when  $n < m$ .

While it is doubtful that this test is biased for large samples, this question will not be considered in the present note.

PRINCETON UNIVERSITY,  
PRINCETON, N. J.

---

## SIGNIFICANCE TEST FOR SPHERICITY OF A NORMAL $n$ -VARIATE DISTRIBUTION

BY JOHN W. MAUCHLY

**1. Introduction.** This note is concerned with testing the hypothesis that a sample from a normal  $n$ -variate population is in fact from a population for which the variances are all equal and the correlations are all zero. A population having this symmetry will be called "spherical." Under a linear orthogonal transformation of variates, a spherical population remains spherical, and consequently the features of a sample which furnish information relevant to this hypothesis must be invariant under such transformations.

A situation for which this test is indicated arises when the sample consists of  $N$   $n$ -dimensional vectors, for which the variates are the  $n$  components along coordinate axes known to be mutually perpendicular, but having an orientation which is, a priori at least, quite arbitrary. A specific application for two dimensions, treated elsewhere [1], may be mentioned. Each of  $N$  days furnishes a sine and a cosine Fourier coefficient for a given periodicity, and these, when plotted as ordinate and abscissa, yield a somewhat elliptical cloud of  $N$  points. The sine and cosine functions are orthogonal, and their variances have