A METHOD FOR RECURRENT COMPUTATION OF ALL THE PRINCIPAL MINORS OF A DETERMINANT, AND ITS APPLICATION IN CONFLUENCE ANALYSIS

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1. Recurrent computation of all the principal minors of a determinant. The formulae which I develop in this paper have been worked out for use in statistical confluence analysis. By means of recurrent computation they shorten considerably the amount of work required to compute all principal minors of a square matrix. Originally I elaborated this method as a simplification of one given by Frisch (not published).

Subsequently I found that the method could more easily be deduced from the pivotal method. This method has been described, for example, by Whittaker and Robinson [5] and by Aitken [1].

Let us consider a square n-rowed matrix

$$\left\| \begin{matrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{matrix} \right\|. \tag{1}$$

Let the adjoint of this matrix be $\| p_{ij} \|$ and let us denote its determinant value by $D_{12\cdots n}$.

Then we have the following identity

$$\begin{vmatrix} p_{n-1,n-1} & p_{n-1,n} \\ p_{n,n-1} & p_{n,n} \end{vmatrix} = D_{12\cdots n} D_{12\cdots n-2}. \tag{2}$$

As Aitken points out, the pivotal method is based upon this identity.

Next consider the following matrix which is formed from the matrix (1) by striking out the nth row and the $(n-1)$th column:

$$\left\| \begin{matrix} a_{11} & \cdots & a_{1,n-2} & a_{1,n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n-2,1} & \cdots & a_{n-2,n-2} & a_{n-2,n} \\ a_{n-1,1} & \cdots & a_{n-1,n-2} & a_{n-1,n} \end{matrix} \right\|. \tag{3}$$