ON THE NON-EXISTENCE OF TESTS OF "STUDENT'S" HYPOTHESIS HAVING POWER FUNCTIONS INDEPENDENT OF σ

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1. Introduction. Consider a system of n random variables $x_1, x_2, \cdots, x_n$ where each is known to be normally distributed about the same but unknown mean, ξ, and with the same, but also unknown standard deviation σ. The assumption, H_0, that ξ has some specified value, ξ_0, e.g. $\xi_0 = 0$, while nothing is assumed about σ, is known as the "Student" Hypothesis. Two aspects of the hypothesis H_0 have been already studied extensively. If the alternatives with respect to which it is desired to test H_0 assume specifically that $\xi > \xi_0$, (or $\xi < 0$), then we have the so-called asymmetric case of "Student's Hypothesis" and it is known, [1], that there exists a uniformly most powerful test of H_0. This consists in the rule, originally suggested by "Student," of rejecting H_0 whenever

$$t = \frac{\bar{x} - \xi_0}{S}\sqrt{n-1} > t_\alpha, \tag{1}$$

where $\bar{x}$ and S denote the mean and the standard deviation of the observed x_i's and t_α is taken, for example, from Fisher's Tables [2] with his $P = 2\alpha$. In other words t_α is such that

$$P\{t > t_\alpha \mid H_0\} = \alpha, \tag{2}$$

where α is the chosen level of significance. In accordance with the definition of the uniformly most powerful test, whenever any other rule, R, offered to test the same hypothesis H_0 has the same probability α of H_0 being rejected when it is true, the power of this alternative test cannot exceed that of "Student's" Test. In other words, if it happens that the true value of ξ is not equal to ξ_0 but is greater, then the probability of this circumstance being detected by "Student's" test is at least equal to that corresponding to the rule R.

If the set of alternative hypotheses is not limited to those specifying the value of ξ either greater or smaller than ξ_0, but includes both those categories, then it is known, [1], that there is no uniformly most powerful test of the hypothesis, H_0. However in this case there exists a slightly different test, also based on "Student's" criterion t, possessing the remarkable property of being unbiased of type B_1, [3]. The test, in common use for a long time, consists in rejecting H_0 when

$$|t| > t_\alpha, \tag{3}$$