THE PRODUCT SEMI-INVARIANTS OF THE MEAN AND A CENTRAL MOMENT IN SAMPLES

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The method developed by the author for calculating the semi-invariants and product semi-invariants of moments in samples from any infinite population[1] is not immediately applicable to the calculation of product semi-invariants of the mean and a central moment in such samples. In the present paper this method is adapted for this purpose so that the calculation of these product semi-invariants becomes routine. As it will be seen, the computing is a little heavier than in the case of central moments alone for results of equal weight. A table of results up to weight ten for the mean and the second, third and fourth central moments is given. The author plans to apply these to a further study of the sampling characteristics of the coefficient of variation and Fisher's t in samples from non-normal populations.

Let a random sample, $x_1, x_2, \cdots, x_N$ of N observations be drawn at random from an infinite population characterized by the semi-invariants, $\lambda_1, \lambda_2, \lambda_3, \cdots$. The sample mean is,

$$\bar{x} = \sum_{i=1}^{N} x_i/N,$$

and the n-th central moment of the sample is

$$m_n = \sum_{i=1}^{N} (x_i - \bar{x})^n/N.$$

Then the product semi-invariants of order kl of x and m_n, $S_{kl}(x, m_n)$, are defined by the formal identity in the parameters ϑ and ω:

$$(1)\quad (S_{10}\vartheta + S_{01}\omega) + \frac{1}{2!}(S_{10}\vartheta + S_{01}\omega)^{(2)} + \frac{1}{3!}(S_{10}\vartheta + S_{01}\omega)^{(3)} + \cdots \equiv \log E(e^{\bar{x}\vartheta + m_n\omega}),$$

in which E denotes the mathematical expectation over the set of all such samples and

$$(S_{10}\vartheta + S_{01}\omega)^{(r)} = \sum_{j=1}^{r} \binom{r}{j} S_{j,r-j}(\bar{x}, m_n)\vartheta^j\omega^{r-j}.$$

[1] "An Application of Thiele's Semi-invariants to the Sampling Problem;" *Metron*, Vol. VII, part IV (1928), pp. 3–75.

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