THE SUBSTITUTIVE MEAN AND CERTAIN SUBCLASSES OF THIS GENERAL MEAN

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1. Introduction. No general agreement has been reached, so far as I know, as to what constitutes a mean. A *necessary condition* which appears to meet with general approval is that a single-valued mean of a set of numbers all equal to a constant c should itself be equal to c. However, there appears to be some valid objection against imposing any *other* proposed condition as *necessary*.

Of course, intermediacy is a condition that suggests itself at once. Indeed, in certain mean value theorems in general analysis—such as the First Theorem of the Mean for integral calculus, which I mention in Section 3—intermediacy is the main feature.

However, O. Chisini [1] insisted that intermediacy or internality is not the chief characteristic of a statistical mean. Rather, a mean is a number to take the place, by substitution, of each of a set of numbers in general different. Such a mean may well be called a *representative* or *substitutive* mean.

Chisini defined m to be a mean of $x_1, x_2, \cdots, x_n$, relative to a function F, provided that

$$F(m, m, \cdots, m) = F(x_1, x_2, \cdots, x_n). \tag{1.1}$$

If, for example,

$$F(x_1, x_2, \cdots, x_n) = \Sigma x_i^2 = \Sigma m^2 = nm^2, \tag{1.2}$$

the mean m thus obtained is the root-mean-square

$$m = \pm [(1/n)\Sigma x_i^2]^{1/2}. \tag{1.3}$$

The choice of F, Chisini noted, depended upon the use to be made of the mean.

Suppose now that $f(x_1, x_2, \cdots, x_n)$ is such a function that one value of

$$f(x, x, \cdots, x) = x. \tag{1.4}$$

And suppose that this f is taken as a particular F for (1.1) to determine a mean m *implicitly*; thus

$$f(m, m, \cdots, m) = f(x_1, x_2, \cdots, x_n). \tag{1.5}$$

Then, from (1.5) and (1.4) it follows that one value of

$$f(x_1, x_2, \cdots, x_n) = m. \tag{1.6}$$

And thus f determines the mean m both *explicitly* and *implicitly*.

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