NOTE ON THEORETICAL AND OBSERVED DISTRIBUTIONS OF REPETITIVE OCCURRENCES

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1. A simple problem of repetitive occurrences. Two questions which the engineer often desires to answer whenever he has a new type of apparatus or a new design of an old type of apparatus are: How many times will it perform its intended function without failure? and How many times will it fail to perform its intended function in a given length of time? To do this, he selects a number of what he believes to be identical units of the apparatus and gives each unit a performance test under a uniform test procedure. The number of satisfactory operations prior to the first observed failure to perform this operation is called a "run" and is a measure of the type desired for each unit.

If it is assumed that the probability of failure at any operation is a constant, q, and the probability of satisfactory operation is $1 - q$ or p, then the mathematical probability of runs of 0, 1, 2, 3 $\cdots$ satisfactory operations for any unit are

$$q, pq, p^2q, p^3q, \cdots \tag{1}$$

respectively.

Let x denote the number of satisfactory operations in any run. The mean value of x, say m_x, is given by

$$m_x = \frac{p}{q}. \tag{2}$$

The variance of x is

$$\sigma_x^2 = \frac{p}{q^2}. \tag{3}$$

The first step in practice is to determine whether there exists a constant probability, p, by means of the application of the operation of statistical control.[1] Expressions (1), (2), and (3) provide the necessary information for doing this. When a constant probability exists as evidenced by at least 25 consecutive samples of 4 units each the following practical procedure has been found to be satisfactory.

1. An estimate of p (or q), the sole parameter of the distribution, can be obtained from the average length of run in the sample. If p is less than 0.6 and if the sample size is large, a reasonably good estimate of p can be obtained from the proportion of the sample having runs of zero length.

2. The probability of getting runs of length x or more is p^x. Thus, if a minimum (or maximum) value of the probability, p^x, is chosen, a maximum

[1] W. A. Shewhart, "*Statistical Method from the Viewpoint of Quality Control*," The Department of Agriculture Graduate School, Washington, 1939, Chapter I.