A STUDY OF A UNIVERSE OF n FINITE POPULATIONS WITH APPLICATION TO MOMENT-FUNCTION ADJUSTMENTS FOR GROUPED DATA

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The object of this paper is to study the case of a universe of n finite populations, considering both the expectations of population moment-functions and the moments of sample moments, and to make applications of the results which may be of interest to mathematical statisticians. The sampling formulas which are derived reduce to the usual infinite or finite sampling formulas, under appropriate assumptions. Also a method is given whereby finite sampling formulas may be transformed into the corresponding infinite sampling formulas.

The general methods and formulas which are given in Part I for the expectations of population moment-functions are used, in Part II, to find the expectations of moments of a distribution of discrete data grouped in "k groupings of k".

I. A Study of a Universe of n Finite Populations

Let ${}_nU_N$ be a universe composed of the set of populations ${}_rX$, $(r = 1, 2, \cdots, n)$ each population ${}_rX$ consisting of a finite number of discrete variates ${}_rx_i$, $(i = 1, 2, \cdots, N)$, $(N > n)$. The tth moment of ${}_rX$ is denoted by ${}_r\mu_t$. The tth central moment of ${}_rX$ is denoted by ${}_r\bar{\mu}_t$. The tth moment and the tth central moment of ${}_nU_N$ are respectively denoted by μ_t and $\bar{\mu}_t$. The expected value of a variable y is denoted by $E(y)$. We have

$$\begin{aligned}
{}_r\mu_t &= E({}_rx_i^t) = \frac{1}{N}\sum_{i=1}^{N} {}_rx_i^t, \qquad {}_r\bar{\mu}_t = E({}_rx_i - {}_r\mu_1)^t = \frac{1}{N}\sum_{i=1}^{N} ({}_rx_i - {}_r\mu_1)^t, \\
\mu_{1:\mu_t} &= E({}_r\mu_t) = \frac{1}{n}\sum_{r=1}^{n} {}_r\mu_t, \qquad \mu_{1:\bar{\mu}_t} = E({}_r\bar{\mu}_t) = \frac{1}{n}\sum_{r=1}^{n} {}_r\bar{\mu}_t, \qquad (1.1)\\
\mu_{s_1s_2\cdots s_v:\mu_{t_1}\mu_{t_2}\mu_{t_v}} &= E({}_{r_1}\mu_{t_1}^{s_1}\,{}_{r_2}\mu_{t_2}^{s_2}\cdots{}_{r_v}\mu_{t_v}^{s_v}), \\
\mu_{s_1s_2\cdots s_v:\bar{\mu}_{t_1}\bar{\mu}_{t_2}\cdots\bar{\mu}_{t_v}} &= E({}_{r_1}\bar{\mu}_{t_1}^{s_1}\,{}_{r_2}\bar{\mu}_{t_2}^{s_2}\cdots{}_{r_v}\bar{\mu}_{t_v}^{s_v}).
\end{aligned}$$

We also note that $\mu_{s_1s_2\cdots s_v:\mu_{t_1}\mu_{t_2}\cdots\mu_{t_v}}$ may be written $\mu_{111\cdots1:t\mu_{t_1}^{s_1}\mu_{t_2}^{s_2}\cdots\mu_{t_v}^{s_v}}$.

1. The expected value of moments and central moments. It follows easily from (1.1) that

$$\mu_{1:\mu_t} = \mu_t. \qquad (1.2)$$