NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

THE STANDARD ERRORS OF THE GEOMETRIC AND HARMONIC MEANS AND THEIR APPLICATION TO INDEX NUMBERS[1]

By Nilan Norris

Attempts to derive useful expressions for estimating the standard deviations of the sampling errors of the geometric and harmonic means have not yielded results comparable with those afforded by the modern theory of estimation, including fiducial inference. There are in the literature of probability theory certain theorems which can be applied to obtain these desired results in a straightforward manner. The use of forms for estimating standard errors is subject to certain conditions which are not always fulfilled, particularly in the case of time series. An understanding of these limitations should deter those who may be tempted to judge the significance of phenomena such as price changes solely on the basis of estimated standard errors of indexes.

1. Statement of formulas. The standard error of the geometric mean of a sequence of positive independent chance variables denoted by $x_i = x_1, x_2, \cdots, x_n$, is $\sigma_G = \theta_1 \frac{\sigma_{\log x}}{\sqrt{n}}$, where θ_1 is the population geometric mean of the variates; so that $\sigma_{\log x}$ is the standard deviation of the logarithms in the population as given by $\sigma_{\log x} = [E\{[\log x - E(\log x)]^2\}]^{\frac{1}{2}}$; and n is the number of individuals comprising the sample. The estimate of the standard error of the geometric mean is $s_G = G \frac{s_{\log x_i}}{\sqrt{n-1}}$, where G is the sample geometric mean, that is, the estimate of θ_1; so that $s_{\log x_i}$ is the estimate of $\sigma_{\log x}$; and $n - 1$ is the degree of freedom of the sample.

[1] This article summarizes two papers presented at sessions of the Institute of Mathematical Statistics at Detroit, Michigan on December 27, 1938, and at Philadelphia, Pennsylvania on December 27, 1939. The results given herein can be derived by several methods, which vary somewhat as to degree of rigor. The writer wishes to acknowledge his indebtedness to the referee for suggesting a proof based on a probability theorem stated by J. L. Doob, "The limiting distributions of certain statistics," *Annals of Math. Stat.*, Vol. 4 (1935), pp. 160–169. The standard deviation formulas obtained follow as an application of this theorem, as will be seen by reference to it. Obviously the asymptotic variance formulas of many other statistics (estimates of parameters) can be obtained in a similar manner.