ON SAMPLES FROM A NORMAL BIVARIATE POPULATION

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1. Introduction. In a number of papers written during the last ten years, J. Neyman and E. S. Pearson[1] have discussed certain general principles underlying the choice of tests of statistical hypotheses. They have suggested that any formal treatment of the subject requires in the first place the specification of (*i*) the hypothesis to be tested, say H_0, (*ii*) the admissible alternative hypotheses. An appropriate test will then consist of a rule to be applied to observational data, for rejecting H_0 in such a way that (*iii*) the risk of rejecting H_0 when it is true is fixed at some desired value (e.g., 0.05 or 0.01), (*iv*) the risk of failing to reject H_0 when some one of the admissible alternatives is true is kept as small as possible. With these general principles in mind, they have investigated how best the condition (*iv*) may be satisfied in different classes of problems. In many cases, though not in all, it has been found that the conditions are satisfied by the test obtained from the use of what has been termed the likelihood ratio, [9], [10], [14]. Once the problem has been specified, the test criterion is usually very easily found, although its sampling distribution, if H_0 is true, often presents great difficulties. In the present paper, I propose to use this method to obtain appropriate tests for a number of hypotheses concerning two normally correlated variables. The investigation was suggested by a recent application of the method by W. A. Morgan [6] to a problem originally discussed by D. J. Finney [3].

2. The hypotheses and the appropriate criteria. A sample of two variables x_1 and x_2 is supposed to have been drawn at random from a normal bivariate population, with the distribution

$$p(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_{12}^2}} \exp\left\{-\frac{1}{2(1-\rho_{12}^2)}\left[\left(\frac{x_1-\xi_1}{\sigma_1}\right)^2 - 2\rho_{12}\left(\frac{x_1-\xi_1}{\sigma_1}\right)\left(\frac{x_2-\xi_2}{\sigma_2}\right) + \left(\frac{x_2-\xi_2}{\sigma_2}\right)^2\right]\right\} \tag{1}$$

where ξ_1, ξ_2, σ_1, σ_2, and ρ_{12} are the population parameters.

Morgan tested the hypothesis that the variances of the two variables are equal, i.e.,

H_1: $\sigma_1 = \sigma_2$.

[1] See bibliography at the end of the paper.