NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

NOTE ON THE ADJUSTMENT OF OBSERVATIONS

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The method of least squares has been extended to the adjustment of observations with errors in more than one variable. The history of the development and its principal results have been given by Deming [2], [3], [4], [5]. The basis is the assumption that for the "best" adjustment the sum of the weighted squares of all the residuals (observed values minus adjusted values) must be made a minimum with respect to the adjustments to the observations and with respect to the parameters involved in the conditions the adjusted values must satisfy. In certain problems, such as some arising in the study of relative growth in biology, this assumption is not adequate; it is necessary that the sum to be minimized be generalized to include cross products as well as squares of the residuals.

Suppose we have a set of n universes of q-dimensional points whose centers of gravity are known to satisfy certain conditions; for instance, they might all lie on a certain type of curve. A sample having been taken from each universe, the center of gravity of each sample is taken as the observed center of gravity of the corresponding universe, and it is desired to determine the most probable set of adjustments to the coordinates and the most probable set of parameters involved in the conditions, subject to the requirement that the adjusted values satisfy the conditions exactly. It is assumed that the sampling distribution of the center of gravity in each universe satisfies the multivariate normal law, and that the standard deviations and coefficients of correlation of each sample may with sufficient accuracy be taken as the constants of the corresponding universe. Then by reasoning analogous to that of the derivation of the least squares principle for one variable from the univariate normal law, the probability of getting the observed set of values is proportional to e^{-Q}, where

$$Q = \sum_{i=1}^{n} Q_i \tag{1}$$

Q_i being a homogeneous quadratic function of the errors at the ith centroid and in general involving the cross products as well as the squares of the errors.