ASYMPTOTICALLY MOST POWERFUL TESTS OF STATISTICAL HYPOTHESES[1]

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1. Introduction. Let $f(x, \theta)$ be the probability density function of a variate x involving an unknown parameter θ. For testing the hypothesis $\theta = \theta_0$ by means of n independent observations $x_1, \cdots, x_n$ on x we have to choose a region of rejection W_n in the n-dimensional sample space. Denote by $P(W_n \mid \theta)$ the probability that the sample point $E = (x_1, \cdots, x_n)$ will fall in W_n under the assumption that θ is the true value of the parameter. For any region U_n of the n-dimensional sample space denote by $g(U_n)$ the greatest lower bound of $P(U_n \mid \theta)$. For any pair of regions U_n and T_n denote by $L(U_n, T_n)$ the least upper bound of

$$P(U_n \mid \theta) - P(T_n \mid \theta).$$

In all that follows we shall denote a region of the n-dimensional sample space by a capital letter with the subscript n.

Definition 1. A sequence $\{W_n\}$, $(n = 1, 2, \cdots, \text{ad inf.})$, of regions is said to be an asymptotically most powerful test of the hypothesis $\theta = \theta_0$ on the level of significance α if $P(W_n \mid \theta_0) = \alpha$ and if for any sequence $\{Z_n\}$ of regions for which $P(Z_n \mid \theta_0) = \alpha$, the inequality

$$\limsup_{n \to \infty} L(Z_n, W_n) \leq 0$$

holds.

Definition 2. A sequence $\{W_n\}$, $(n = 1, 2, \cdots, \text{ad inf.})$, of regions is said to be an asymptotically most powerful unbiased test of the hypothesis $\theta = \theta_0$ on the level of significance α if $P(W_n \mid \theta_0) = \lim_{n=\infty} g(W_n) = \alpha$, and if for any sequence $\{Z_n\}$ of regions for which $P(Z_n \mid \theta_0) = \lim_{n=\infty} g(Z_n) = \alpha$, the inequality

$$\limsup_{n \to \infty} L(Z_n, W_n) \leq 0$$

holds.

Let $\hat{\theta}_n(x_1, \cdots, x_n)$ be the maximum likelihood estimate of θ in the n-dimensional sample space. That is to say, $\hat{\theta}_n(x_1, \cdots, x_n)$ denotes the value of θ

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