

the test? For alternative hypothesis  $H_4$  ( $p_1 = .75$ ), and for  $P > .90$ , referring to Table I, it is seen that 42 paired samples must be employed and not more than 14 may be placed incorrectly. Under the same alternative hypothesis, if it be required merely that  $P > .50$  (i.e., an expert with an ability of .75 have better than an even chance of passing), then only 18 paired samples are necessary and not more than 4 may be arranged incorrectly.

Thus, before conducting an experiment in which the Sign Test is to be employed, if the experimenter first decides what power the test must have relative to a certain alternative hypothesis; then from the accompanying table he may learn the minimum number of paired samples that are necessary; and the related maximum value of  $r$ .

If this procedure is not followed, and an experimenter employs, say 6 paired samples, he may (as can be seen from the table) discover, to his dismay, that "experts" of ability .75 will be unrecognized more than 80% of the time.

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**MOMENTS OF THE RATIO OF THE MEAN SQUARE SUCCESSIVE  
DIFFERENCE TO THE MEAN SQUARE DIFFERENCE IN  
SAMPLES FROM A NORMAL UNIVERSE**

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The following result may have considerable application to trend analysis. The specific problem was proposed to me by R. H. Kent.

Consider a sample  $0_n : X_1, X_2, \dots, X_n$  from a normal population with zero mean and variance  $\sigma^2$ , the variates being arranged in temporal order. We seek the moments of the ratio of  $\delta^2$  to  $S^2$ , where

$$(1) \quad (n-1)\delta^2 = \sum_{j=1}^{n-1} (X_j - X_{j+1})^2$$

and

$$(2) \quad nS^2 = \sum_{j=1}^n (X_j - \bar{X})^2.$$

Here  $\bar{X}$  is the mean of the  $X_j$ . In order to simplify the algebra, we will work with quantities  $A$  and  $B$  defined by

$$(3) \quad \begin{aligned} 2\sigma^2 A &= (n-1)\delta^2, \\ 2\sigma^2 B &= nS^2. \end{aligned}$$

The characteristic function for the joint distribution of  $A$  and  $B$  is

$$(4) \quad \begin{aligned} \varphi(t_1, t_2) &= E(e^{At_1 + Bt_2}) \\ &= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \exp\left(At_1 + Bt_2 - \frac{1}{2\sigma^2} \sum_{j=1}^n X_j^2\right) \prod_{j=1}^n dX_j, \end{aligned}$$