NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

NOTE ON THE DISTRIBUTION OF NON-CENTRAL t WITH AN APPLICATION

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If we adopt the notation recently used by N. L. Johnson and B. L. Welch [1], non-central t is defined by

$$t = \frac{z + \delta}{\sqrt{w}},$$

in which δ is a constant and z and w are independent variables, z being distributed normally about zero with unit variance and w being distributed as χ^2/f in which f is the number of degrees of freedom for χ^2.

In the paper referred to Johnson and Welch discuss some applications of non-central t and give suitable tables calculated from the probability integral of the distribution of this variable. Previously tables of this probability integral for the purpose of calculating the power of the t test had been given by J. Neyman [2] and Neyman and B. Tokarska [3].

It is the purpose of this note to call attention to a series expansion for the probability integral of non-central t which is simple in form and in most cases convenient for direct calculation. As an application of some intrinsic interest this series is used to compute in several numerical cases the power of a test proposed by E. J. G. Pitman [4] based on the randomization principle.

If for convenience we write,

$$\sqrt{w} = \psi, \ (0 \leq \psi \leq \infty),$$

we have for the joint distribution of $z + \delta$ and ψ,

$$(1) \qquad df(z + \delta, \psi) = \frac{2(f/2)^{f/2}}{\sqrt{2\pi}\,\Gamma(f/2)} e^{-\frac{1}{2}(f\psi^2+z^2)} \psi^{f-1}\, d\psi\, dz.$$

From this

$$(2) \qquad \begin{aligned} df(t, \psi) &= \frac{2(f/2)^{f/2} e^{-\delta^2/2}}{\sqrt{2\pi}\,\Gamma(f/2)} e^{-\psi^2(f+t^2)/2+\delta\psi t} \psi^f\, d\psi\, dt \\ &= \frac{2(f/2)^{f/2} e^{-\delta^2/2}}{\sqrt{2\pi}\,\Gamma(f/2)} e^{-\psi^2(f+t^2)/2} \sum_{r=0}^{\infty} \frac{(\delta t)^r}{r!} \psi^{f+r}\, d\psi\, dt, \end{aligned}$$