THE MEAN SQUARE SUCCESSIVE DIFFERENCE

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1. Introduction. In making measurements, every precaution is generally taken to hold the conditions of the experiment constant, in order that the population, whose parameters are to be estimated from the observations, shall remain fixed throughout the experiment. One wishes each observation to come from the same population, or what is the same thing if normality is assumed, from populations having the same means and standard deviations.

There are cases, however, where the standard deviation may be held constant, but the mean varies from one observation to the next. If no correction is made for such variation of the mean, and the standard deviation is computed from the data in the conventional way, then the estimated standard deviation will tend to be larger than the true population value. When the variation in the mean is gradual, so that a trend (which need not be linear) is shifting the mean of the population, a rather simple method of minimizing the effect of the trend on dispersion is to estimate standard deviation from differences. It is for this purpose that the mean square successive difference

\[ \delta^2 = \frac{\sum_{i=1}^{n-1} (x_{i+1} - x_i)^2}{n - 1} \]

is suggested. The subscript \(i\) in this expression refers to the temporal order of the observation \(x_i\).

In using \(\delta^2\) for estimating standard deviation, the distribution of \(\delta^2\) in random samples is of interest, since questions of bias, efficiency, and confidence interval require consideration. \(\delta^2\) may be used, in addition, to determine whether a trend actually exists; in this case one must know whether \(\delta^2\) differs significantly from

\[ s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}, \]

which measures variance independently of the order of the observations, and consequently includes the effect of the trend.

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