THE CYCLIC EFFECTS OF LINEAR GRADUATIONS PERSISTING IN THE DIFFERENCES OF THE GRADUATED VALUES

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1. Scope of inquiry. Slutzky [1] applied the moving sum, the repeated moving sum, and other linear processes to random numbers obtained from lottery drawings. But the graph of the *moving sum* becomes, when the vertical scale is changed in the ratio of n to 1, the graph of the *moving average*, the simplest form of *graduation*. When cyclic effects are studied, there is no essential difference between a moving sum and a moving average, nor between a general linear process with coefficients $a_1, a_2, \cdots, a_s$, having sum $A \neq 0$ and the corresponding *graduation*, with coefficients $a_i' = a_i/A$. Thus Slutzky's work throws considerable light upon graduation, although his main interest was in summation.

Slutzky found that the graphs of moving sums of random numbers bore strong resemblance to graphs of economic phenomena, such as [1, p. 110] that of English business cycles from 1855 to 1877. In fact, Slutzky regards the fluctuations in economic phenomena as due largely to a synthesizing of random causes.

In general the undulatory character of such values cannot be described as periodic; since the waves are of different length. But Slutzky found that, upon operating on random data having mean zero and constant variance, the resulting values approach a sinusoidal limit under certain conditions,—in particular, when a set of n summations by twos is followed by m differencings, and as $n \to \infty$, $m/n \to$ a constant. Romanovsky [2] generalized this result by taking successive summations of s consecutive elements of the data, with $s \geqq 2$; but required that $m/n \to \alpha \neq 1$. However, the cases which are of interest to me just now are those for which $m = n - 1$ or $m = n - 2$; and for these cases $m/n \to 1$. Romanovsky considers the case of $m = n - 1$,—not, however, as leading to a sinusoidal limit,—and gives in formula (46) the value of a coefficient of correlation—which I deduce directly. From his formula (43) a corresponding coefficient of correlation can be obtained for the case of $m = n - 2$, as the sum of certain products. A more simple expression than this I need, which I obtain directly. In my treatment, these coefficients are the cosines of angles; and the ratio of such an angle to a whole revolution is an expected frequency of occurrence.

After setting forth in Section 2 some preliminary formulas, I treat in Section 3 the results of applying to random data an indefinite number $k + 2$ of summations or averagings, followed by k differencings—the number of terms in a sum remaining fixed. In Section 4, however, only a few differencings are applied to a