ON THE PROBABILITY OF THE OCCURRENCE OF AT LEAST m EVENTS AMONG n ARBITRARY EVENTS

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Introduction. Let $E_1, \cdots, E_n$, denote n arbitrary events. Let $p_{\nu_1' \cdots \nu_i' \nu_{i+1} \cdots \nu_j}$, where $0 \leqq i \leqq j \leqq n$ and $(\nu_1, \cdots, \nu_j)$ is a combination of the integers $(1, \cdots, n)$, denote the probability of the non-occurrence of $E_{\nu_1}, \cdots, E_{\nu_i}$ and the occurrence of $E_{\nu_{i+1}}, \cdots, E_{\nu_j}$. Let $p_{[\nu_1 \cdots \nu_i]}$ denote the probability of the occurrence of $E_{\nu_1}, \cdots, E_{\nu_i}$ and no others among the n events. Let $S_j = \Sigma p_{\nu_1 \cdots \nu_j}$ where the summation extends to all combinations of j of the n integers $(1, \cdots, n)$. Let $p_m(\nu_1, \cdots, \nu_k)$, $(1 \leqq m \leqq k \leqq n)$, denote the probability of the occurrence of at least m events among the k events $E_{\nu_1}, \cdots, E_{\nu_k}$.

By the set $(x_1, \cdots, x_b, \cdots, x_a) - (x_1, \cdots, x_b)$ (where $b \leqq a$) we mean the set $(x_{b+1}, \cdots, x_a)$. And by a $\binom{a}{b}$-combination out of $(x_1, \cdots, x_a)$ we mean a combination of b integers out of the a integers $(x_1, \cdots, x_a)$.

We often use summation signs with their meaning understood, thus for a fixed k, $1 \leqq k \leqq n$, the summations in $\Sigma p_{\nu_1 \cdots \nu_k}$, or $\Sigma p_m(\nu_1, \cdots, \nu_k)$, extend to all the $\binom{n}{k}$-combinations out of $(1, \cdots, n)$.

The following conventions concerning the binomial coefficients are made:

$$\binom{0}{0} = 1, \qquad \binom{a}{b} = 0 \qquad \text{if} \qquad a < b \qquad \text{or if} \qquad b < 0.$$

It is a fundamental theorem in the theory of probability that, if $E_1, \cdots, E_n$ are incompatible (or "mutually exclusive"), then

$$p_1(1, \cdots, n) = p_1 + \cdots + p_n .$$

When the events are arbitrary, we have Boole's inequality

$$p_1(1, \cdots, n) \leqq p_1 + \cdots + p_n .$$

Gumbel[1] has generalized this inequality to the following:

$$p_1(1, \cdots, n) \leqq \frac{\Sigma p_1(\nu_1, \cdots, \nu_k)}{\binom{n-1}{k-1}},$$

[1] *C. R. Acad. Sc.* Vol. 205(1937), p. 774.