DISTRIBUTION OF THE RATIO OF THE MEAN SQUARE SUCCESSIVE DIFFERENCE TO THE VARIANCE

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1. Introduction. Let $x_1, \ldots, x_n$ be variables representing $n$ successive observations in a population which obeys a distribution law

$$ce^{-(x-\xi)^2/2\sigma^2}dx,$$

$$c = \frac{1}{\sigma \sqrt{2\pi}},$$

i.e. which is normal, with the mean $\xi$ and the standard deviation $\sigma$. For the sample we define as usual the mean,

$$\bar{x} = \frac{1}{n}\sum_{\mu=1}^{n} x_\mu,$$

the variance,

$$s^2 = \frac{1}{n}\sum_{\mu=1}^{n} (x_\mu - \bar{x})^2,$$

and also the mean square successive difference

$$\delta^2 = \frac{1}{n-1}\sum_{\mu=1}^{n-1} (x_{\mu+1} - x_\mu)^2.$$

The reasons for the study of the distribution of the mean square successive difference $\delta^2$, in itself as well as in its relationship to the variance $s^2$, have been set forth in a previous publication, to which the reader is referred. The distribution of $\delta^2$, and in particular its moments, were also studied there. The present paper is devoted to the investigation of the ratio

$$\eta = \frac{\delta^2}{s^2}.$$

A comparison of the observed value of $\eta$ with that distribution is particularly suited as a basis of the judgment whether the observations $x_1, \ldots, x_n$ are independent or whether a trend exists. (Cf. sections 1 and 2, loc. cit.\(^5\))

The moments of $\eta$ have already been determined by J. D. Williams by a

\(^1\) Also Scientific Advisory Committee of the Ballistic Research Laboratory, Aberdeen Proving Ground.


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