DISTRIBUTION OF THE RATIO OF THE MEAN SQUARE SUCCESSIVE DIFFERENCE TO THE VARIANCE

By John von Neumann

Institute for Advanced Study[1]

1. Introduction. Let $x_1, \cdots, x_n$ be variables representing n successive observations in a population which obeys a distribution law

$$ce^{-(x-\xi)^2/2\sigma^2}\,dx, \qquad \left(c = \frac{1}{\sigma\sqrt{2\pi}}\right),$$

i.e. which is normal, with the mean ξ and the standard deviation σ. For the sample we define as usual the mean,

$$\bar{x} = \frac{1}{n}\sum_{\mu=1}^{n} x_\mu,$$

the variance,

$$s^2 = \frac{1}{n}\sum_{\mu=1}^{n}(x_\mu - \bar{x})^2,$$

and also the mean square successive difference

$$\delta^2 = \frac{1}{n-1}\sum_{\mu=1}^{n-1}(x_{\mu+1} - x_\mu)^2.$$

The reasons for the study of the distribution of the mean square successive difference δ^2, in itself as well as in its relationship to the variance s^2, have been set forth in a previous publication[2], to which the reader is referred. The distribution of δ^2, and in particular its moments, were also studied there. The present paper is devoted to the investigation of the ratio

$$\eta = \frac{\delta^2}{s^2}.$$

A comparison of the observed value of η with that distribution is particularly suited as a basis of the judgment whether the observations $x_1, \cdots, x_n$ are independent or whether a trend exists. (Cf. sections 1 and 2, loc. cit.[2])

The moments of η have already been determined by J. D. Williams by a

---

[1] Also Scientific Advisory Committee of the Ballistic Research Laboratory, Aberdeen Proving Ground.

[2] John von Neumann, R. H. Kent, H. R. Bellinson, B. I. Hart, "The mean square successive difference," *Annals of Math. Stat.*, Vol. 12 (1941), pp. 153–162.