3. **The special case.** Let us now return to the special case mentioned at the end of 1—the application to the mean square successive difference.

There $p = 1$ and $B = (0)$, so that the "distribution" of ζ is concentrated at the point 0. Hence $\omega_B(\zeta)$ is an "improper" distribution, concentrated in the same way.[3] Using C and A as described at the end of 1, the above formula becomes (now $m = n - 2$, $p = 1$)

$$\text{(II)} \qquad \omega_{A+(0)}(\theta) = \frac{\Gamma[\frac{1}{2}(n-1)]}{\Gamma[\frac{1}{2}(n-2)]\Gamma[\frac{1}{2}]} \int_0^1 d\rho \cdot \omega_A\left(\frac{\theta}{\rho}\right) \rho^{\frac{1}{2}n-3}(1-\rho)^{-\frac{1}{2}}.$$

It would have been equally easy, of course, to establish (II) directly.

Putting $\rho = 1/t$ gives

$$\text{(III)} \qquad \omega_{A+(0)}(\theta) = \frac{\Gamma[\frac{1}{2}(n-1)]}{\Gamma[\frac{1}{2}(n-2)]\Gamma[\frac{1}{2}]} \int_1^\infty dt \cdot \omega_A(\theta t) t^{-\frac{1}{2}(n-3)}(t-1)^{-\frac{1}{2}}.$$

Since $\omega_A(\gamma)$ vanishes for $|\gamma| > \cos(\pi/n)$, we may replace this integral $\int_1^\infty$ by $\int_1^{\cos(\pi/n)/|\theta|}$

Formula (III) can be used for numerical work, and also to extend the formula (3) on p. 391, loc. cit., to even values of n.

CONVEXITY PROPERTIES OF GENERALIZED MEAN VALUE FUNCTIONS

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In an article appearing in the *Annals of Mathematical Statistics*[1] it was pointed out that while the mean value functions appearing below have been studied and used since 1840, there appeared to have been no attempt made to investigate the behavior of their second derivatives.

Consider (1) the unit weight or simple sample form

$$\varphi(t) \equiv \left(\frac{x_1^t + x_2^t + \cdots + x_n^t}{n}\right)^{1/t},$$

in which the x_i are positive numbers and in which t may take any real value; (2) the weighted sample form

$$\omega(t) \equiv \left(\frac{c_1 x_1^t + c_2 x_2^t + \cdots + c_n x_n^t}{c_1 + c_2 + \cdots + c_n}\right)^{1/t},$$

[3] Dirac's famous "delta function." It could be described by a Stieltjes integral.

[1] Nilan Norris, "Convexity properties of generalized mean value functions," *Annals of Math. Stat.*, Vol. 8 (1937), pp. 118-120.